Key Points: Simultaneous Equations

Solving Linear Simultaneous Equations by Elimination

This method involves combining two equations with two unknowns to form one equation with one unknown.

Example:

$$4x + 3y = 45$$

$$3x - 2y = 4$$

The steps required are:

Label each equation (1) and (2).
Multiply one or both equations by a number so that the number in front of the term we are trying to eliminate is the same but of opposite sign in each equation to form a new pair of equations. So to eliminate y :

$4x + 3y = 45$ (1) multiply by 2
$3x - 2y = 4$ (2) multiply by 3

$8x + 6y = 90$ (3) this is the result of (1) x 2
$9x - 6y = 12$ (4) this is the result of (2) x 3

Add equations (3) and (4) and solve the equation that is left:

$$17x = 102$$
$$\therefore x = 6$$

Substitute this value into either of the original equations and solve the equation that results:

$4(6) + 3y = 45$ (substituting $x = 6$ into (1))
$\therefore 24 + 3y = 45$
$\therefore 3y = 21$
$\therefore y = 7$

Hence the solution to the given simultaneous equation is $x = 6$ and $y = 7$.

Solving Linear Simultaneous Equations by Substitution

This method involves taking an expression for one of the unknowns and substituting it into the other equation which can then be solved.

Example:

$$x - 3y = 2$$
$$3x + 4y = 32$$

The steps required are:

Label each equation (1) and (2):

$x - 3y = 2$ (1)

$$3x + 4y = 32 \qquad (2)$$

Make either x or y the subject of either equation:

$$x = 2 + 3y \qquad \text{(by rearranging (1))}$$

Substitute the resulting expression into the other equation and solve the resulting equation:

$$3(2 + 3y) + 4y = 32 \qquad \text{(by substituting into (2))}$$
$$\therefore 6 + 9y + 4y = 32$$
$$\therefore 13y = 26$$
$$\therefore y = 2$$

Substitute this value back into either of the original equations and solve the resulting equation:

$$x - 3(2) = 2 \qquad \text{by substituting } y = 2 \text{ into (1)}$$
$$\therefore x = 8$$

Key Points: Simultaneous Equations

Solving Simultaneous Equations (Linear and Quadratic)

To solve simultaneous equations in which one is quadratic and one is linear relies upon solving by substituting. The steps to take are similar to those taken for solving linear simultaneous equations by substitution. It is possible to have either one, two or no sets of solutions.

Example:

$$6x + y = 12 \qquad (1)$$

$$x^2 - 3x - y = -2 \qquad (2)$$

The steps required are:

Label each equation (1) and (2) if required. Rearrange the linear equation to make one of the unknowns the subject:

$$y = 12 - 6x \qquad \text{(by rearranging (1))}$$

Substitute the expression obtained into the quadratic expression to obtain a quadratic expression with one unknown:

$$x^2 - 3x - (12 - 6x) = -2$$

$$x^2 + 3x - 10 = 0$$

Solve the resulting quadratic equation by factorising or completing the square or the quadratic formula:

$$(x + 5)(x - 2) = 0$$

$$\therefore x = -5 \text{ or } x = 2$$

Place each of these values in turn into the linear equation to obtain the results:

$$6(-5) + y = 12 \qquad \text{so when } x = -5,\ y = 42$$

Or

$$6(2) + y = 12 \qquad \text{so when } x = 2,\ y = 0.$$

Each pair of answers can be checked by substituting the values into the original equations.