

Indices: Problem-solving

1. Solve the following mathematical equation to get the value of y :

$$(x^3)^y = \frac{(x^y)^y}{x^2}$$

2. Solve the following simultaneous equations to get the values of a and b :

$$(x^a)^2 = (x^4)^{2b} \quad \text{and} \quad (\sqrt{x})^a = \frac{x^3}{x^b}$$

3. Given that $(\sqrt{x^b})^a = x^a \times x^b$ find an expression for a in terms of b :

4. Solve the following equation to obtain the value of a :

$$\frac{x \times x^4 \times x^a}{\sqrt{x}} = \sqrt{\frac{(x^a)^8}{\left(\frac{1}{x^3}\right)}}$$

Indices: Problem-solving Solutions

1. Solve the following mathematical equation to get the value of y :

$$\begin{aligned}(x^3)^y &= \frac{(x^y)^y}{x^2} \\ x^{3y} &= x^{y^2-2} \\ 3y &= y^2 - 2 \\ y^2 - 3y - 2 &= 0 \\ y &= \frac{3 \pm \sqrt{9+8}}{2} = \frac{3 \pm \sqrt{17}}{2}\end{aligned}$$

2. Solve the following simultaneous equations to get the values of a and b :

$$\begin{aligned}(x^a)^2 &= (x^4)^{2b} & \text{and} & & (\sqrt{x})^a &= \frac{x^3}{x^b} \\ x^{2a} &= x^{8b} \Rightarrow 2a = 8b \Rightarrow a = 4b \\ x^{\frac{1}{2}a} &= x^{3-b} \Rightarrow \frac{a}{2} = 3-b \Rightarrow a = 6-2b \\ 6-2b &= 4b \Rightarrow 6b = 6 \Rightarrow b = 1 \\ a &= 4 \times 1 = 4 \\ a &= 4, b = 1\end{aligned}$$

3. Given that $(\sqrt{x^b})^a = x^a \times x^b$ find an expression for a in terms of b :

$$\begin{aligned}x^{\frac{ab}{2}} &= x^{a+b} \Rightarrow \frac{ab}{2} = a+b \\ ab &= 2a+2b \\ a(b-2) &= 2b \\ a &= \frac{2b}{b-2}\end{aligned}$$

4. Solve the following equation to obtain the value of a :

$$\begin{aligned}\frac{x \times x^4 \times x^a}{\sqrt{x}} &= \sqrt{\frac{(x^a)^8}{\left(\frac{1}{x^4}\right)}} \\ x^{5+a-\frac{1}{2}} &= \left(\frac{x^{8a}}{x^{-4}}\right)^{\frac{1}{2}} \Rightarrow x^{5+a-\frac{1}{2}} = x^{4a+2} \\ \frac{9}{2} + a &= 4a + 2 \\ 3a &= \frac{5}{2} \Rightarrow a = \frac{5}{6}\end{aligned}$$