

## Question Sheet: Simultaneous Equations

### Question 1

Solve the simultaneous equations:

$$8x + 6y = -21$$

$$-4x + 10y = 17$$

### Question 2

The line  $y = 5x + 11$  meets the parabola  $y = 2x^2 + 4x + 1$  in two places.

- Show that the  $x$  ordinates of the points of intersection satisfy the equation  $2x^2 - x - 10 = 0$
- Solve the equation  $2x^2 - x - 10 = 0$
- Find the coordinates of the points of intersection.

### Question 3

The line  $y = 6x$  meets the parabola  $y = x^2 + 4x + 1$ .

- Show that the  $x$  ordinates of any points of intersection satisfy the equation  $x^2 - 2x + 1 = 0$
- Show that the determinant of this quadratic equation equals zero
- Give a geometric interpretation of this result?

### Question 4

Solve the simultaneous equations:

$$x + 3y = -1 \text{ and } 2x^2 + 3y^2 = 11$$

### Question 5

Find possible values of  $k$  if the line  $y = 3x + k$  meets the parabola  $y = x^2 + 10x + 13$  at two distinct real points.

## Question Sheet: Simultaneous Equations

### Question 1

Solve the simultaneous equations:

$$8x + 6y = -21$$

$$-4x + 10y = 17$$

### Solution

$$8x + 6y = -21$$

$$-8x + 20y = 34$$

$$\therefore 26y = 13$$

$$\therefore y = 0.5$$

Using the first equation with this value of  $y$  gives  $x = -3$

### Question 2

The line  $y = 5x + 11$  meets the parabola  $y = 2x^2 + 4x + 1$  in two places.

- Show that the  $x$  ordinates of the points of intersection satisfy the equation  $2x^2 - x - 10 = 0$
- Solve the equation  $2x^2 - x - 10 = 0$
- Find the coordinates of the points of intersection.

### Solution

a)

$$2x^2 + 4x + 1 = 5x + 11$$

$$\therefore 2x^2 - x - 10 = 0$$

b)

$$(2x - 5)(x + 2) = 0$$

$$\therefore x = 2.5, x = -2$$

c) Using  $x = 2.5$  gives  $y = 23.5$

Using  $x = -2$  gives  $y = 1$

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### Question 3

The line  $y = 6x$  meets the parabola  $y = x^2 + 4x + 1$ .

- Show that the  $x$  ordinates of any points of intersection satisfy the equation  $x^2 - 2x + 1 = 0$
- Show that the determinant of this quadratic equation equals zero
- Give a geometric interpretation of this result?

### Solution

- $6x = x^2 + 4x + 1$   
 $\therefore x^2 - 2x + 1 = 0$
- $b^2 - 4ac = (-2)^2 - 4(1)(1) = 0$
- $y = 6x$  is a tangent to the curve  $y = x^2 + 4x + 1$

### Question 4

Solve the simultaneous equations:

$$x + 3y = -1 \text{ and } 2x^2 + 3y^2 = 11$$

### Solution

$$\begin{aligned} 2(-1 - 3y)^2 + 3y^2 &= 11 \\ \therefore 2(1 + 3y)^2 + 3y^2 &= 11 \\ \therefore 2 + 12y + 12y^2 &= 11 \\ \therefore 4y^2 + 4y - 3 &= 0 \\ \therefore (2y - 1)(2y + 3) &= 0 \\ \therefore y = 0.5, y = -1.5 \end{aligned}$$

If  $y = 0.5$  then  $x = -2.5$  and if  $y = -1.5$  then  $x = 3.5$

### Question 5

Find possible values of  $k$  if the line  $y = 3x + k$  meets the parabola  $y = x^2 + 10x + 13$  at two distinct real points.

### Solution

$$\begin{aligned} 3x + k &= x^2 + 10x + 13 \\ \therefore x^2 + 7x + 13 - k &= 0 \end{aligned}$$

Require discriminant to be positive, so require  $7^2 - 4(13 - k) > 0$  which occurs when  $k > \frac{3}{4}$