| Surname | Other names |
|---|---|
| | |

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Centre Number | Candidate Number

# Algebraic proof

## Non-Calculator

**Higher Tier**

Assessment Materials – Issue

**Time: n/a**

Paper Reference
**1MA1/1H**

**You must have:** Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

Total Marks

## Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
- *– there may be more space than you need.*
  **Calculators may not be used.**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out**.

## Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
  *– use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

S48572A

*S48572A0120*

*Turn over*

PEARSON


6/4/7/7/4/6/6/

**Answer ALL questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

**1.** Prove that the sum of any odd number and any even number is odd.

$2n+1$ $\quad$ $2n \leftarrow$ even.

$2n+1+2n = 4n+1$

$4n+1$

even so $+1$ = odd

**(Total for Question is 2 marks)**

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**2.** Prove that half the sum of four consecutive numbers is odd.

$n \quad n+1 \quad n+2 \quad n+3$

Sum $= 4n+6$

$\frac{1}{2}$ Sum $= 2n+3$

$2n+3$

even so $+3$ = odd.

**(Total for Question is 3 marks)**

**3.** Prove that the product of any odd number and any even number is even.

$2n$ $\quad$ $2n+1$

even $\quad$ odd

$2n \times (2n+1) = 4n^2 + 2n$

$2(2n^2+n)$

multiple of 2 = even.

**(Total for Question is 3 marks)**

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**4.** Show that the sum of any three consecutive multiples of 3 is also a multiple of 3.

$3n \quad 3n+3 \quad 3n+6$

Sum = $9n + 12$

Factaise → $3(3n+4)$.

multiple of 3

**(Total for Question is 3 marks)**

**5.** Consider the sequence: 4, 7, 10, 13, 16

Prove that the product of any two terms of this sequence is also a term of the sequence.

$n^{th}$ term $= 3n+1$

Other term $= 3m+1$

Product $= (3n+1)(3m+1)$

$= 9mn + 3m + 3n + 1$

Factorise $3(mn + m + n) + 1$

which is a multiple of $3n+1$ (the $n^{th}$ term)

**(Total for Question is 4 marks)**

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**6.** Prove that $(n + 1)^2 - (n - 1)^2 + 1$ is always odd for all positive integer values of n.

$(n+1)^2 = n^2 + 2n + 1$

$(n-1)^2 = n^2 - 2n + 1$ −

$4n$

$4n + 1$

even as multiple of 2 so $+1 =$ ODD

**(Total for Question is 3 marks)**

**7.** Prove that $(5n + 1)^2 - (5n - 1)^2$ is a multiple of 5 for all positive integer values of n.

$(5n+1)^2 = 25n^2 + 10n + 1$

$(5n-1)^2 = 25n^2 - 10n + 1 \quad -$

$20n$

$5(4n)$

multiple of 5!

**(Total for Question is 3 marks)**

**8.** Show that

$$(2a - 1)^2 - (2b - 1)^2 = 4(a - b)(a + b - 1).$$

$(2a-1)^2 = 4a^2 + 4a + 1$

$(2b-1)^2 = 4b^2 + 4b + 1 \quad -$

$-4b^2$

$4a^2 \cancel{-4b} + 4a - \cancel{B}4b$

$4(a^2 - b^2 + a + b)$

$4(a-b)(a+b-1).$

**(Total for Question is 4 marks)**

**9.** Prove that the square of any odd number is always one more than a multiple of 8.

$(2n+1)^2 = 4n^2+4n+1$

$= 4[n^2+n]+1$

multiple of 4 ∴ multiple of 8.

So +1 will always = 1 more than multiple of 8.

**(Total for Question is 4 marks)**

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**10.** Show algebraically that the sum of any three consecutive even numbers is always a multiple of 6.

$2n \quad 2n+2 \quad 2n+4$

Sum $= 6n+6$

$\Rightarrow 6(n+1)$

multiple of 6.

**(Total for Question is 3 marks)**

**11.** Prove that $(2n + 1)^2 - (2n - 1)^2 - 10$ is not a multiple of 8 for all positive integer values of n

$(2n+1)^2 = 4n^2 + 4n + 1$

$(2n-1)^2 = 4n^2 - 4n + 1$ $-$

$8n$

$8n - 10$

Cannot factorise for 8

$\therefore$ NOT a multiple of 8!

**(Total for Question is 4 marks)**

**12.** Prove algebraically that for any two numbers, the product of their difference and their sum is equal to the difference of their squares.

$$(2m - 2n) \times (2m + 2n)$$

$$= 4m^2 + 4mn - 4mn - 4n^2$$

$$= 4m^2 - 4n^2$$

$$(2m)^2 - (2m)^2 =$$

$$= 4m^2 - 4n^2$$

$$\underline{4m^2 - 4n^2 = 4m^2 - 4n^2} \ !$$

**(Total for Question is 4 marks)**

**13.** Prove algebraically that the sum of the squares of any three consecutive even numbers is always a multiple of four.

$(2n)^2 \quad (2n+2)^2 \quad (2n+4)^2$

$4n^2 \quad 4n^2+8n+4 \quad 4n^2+16n+16$

Sum $= 12n^2+24n+20$

Factaise $4(3n^2+6n+5)$

multiple of ~~8~~ 4.

**(Total for Question is 4 marks)**

**14.** Prove algebraically that the sum of the squares of any two consecutive numbers always leaves a remainder of 1 when divided by 4.

$(2n)^2 + (2n+1)^2$

$4n^2 + 4n^2 + 4n + 1$

$8n^2 + 4n + 1$

$4[2n^2 + n] + 1$

multiple of 4

An extra 1 !

**(Total for Question is 4 marks)**