AQA Realising potential

TOPIC TEST

A-level
FURTHER MATHS

Further Vectors & Work, energy and power

Specification content coverage: F1, F3, F4, F6, MC1, MC2, MC3, MC4, MC7

In this test you will be assessed on:

- understanding and using the vector and Cartesian forms of a straight line in three dimensions
- understanding and using the vector and Cartesian forms of a plane in three dimensions
- understanding and using the scalar product to find angles between lines and planes
- finding the perpendicular distance between a line and a plane and the perpendicular distance from a point to a line
- finding intersection points.

- finding the work done by a force acting in the direction of motion or directly opposing the motion
- use of gravitational potential energy and kinetic energy in conservation of energy problems
- use of Hooke's Law including use of the modulus of elasticity.

The test comprises four sections.
The questions in section A will test you on the basics of the Further Vectors topic. Those in section B are Further Vector questions requiring a bit more thinking.
The questions in section A will test you on the basics of the Work, energy and power topic. Those in section B are Work, energy and power questions requiring a bit more thinking.

Section A: The basics

1 (a) Find the vector equation of the line through the points $(2,3,-1)$ and $(1,-1,0)$

[2 marks]

1 (b) Show that the line in **(a)** and **(b)** intersects with the line $\mathbf{r} = \begin{pmatrix} -6 \\ -2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$ and find the point of intersection.

[3 marks]

2 Find the acute angle between the lines

$$\mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} 1 \\ 6 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$$

Give your answer to three significant figures.

[2 marks]

3 The line l has equation $\frac{x-1}{2} = \frac{y-1}{-2} = \frac{z+3}{-1}$

The point A has coordinates $(1,2,-1)$

3 (a) Write down the coordinates of a general point, P, on the line l, using t as the parameter.

[1 mark]

3 (b) Find the vector $\overrightarrow{AP}$ in terms of t

[1 mark]

3 (c) Hence, find the shortest distance between the point A and the line l

[4 marks]

Section B: A bit more thinking

4 The points X and Y have position vectors $\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $2\mathbf{i} + \mathbf{j} - \mathbf{k}$ respectively relative to a fixed origin O.

4 (a) Use a vector method to find the exact value of $\cos OXY$

[2 marks]

4 (b) Hence, find the area of triangle OXY in the form $p\sqrt{q}$ where p, q are rational numbers.

[2 marks]

5 The path of a comet is modelled by the line

$$\mathbf{r} = \begin{pmatrix} 7 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

where one unit represents a distance of one million kilometres.

Earth is positioned at the origin in this model.

5 (a) Find the coordinates of the point of closest approach.

[3 marks]

5 (b) Determine whether the comet ever comes within 5 million kilometres of Earth.

[1 mark]

6 Show that the shortest distance between the lines with equations

$$\mathbf{r}_1 = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + s\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ and } \mathbf{r}_2 = \begin{pmatrix} 9 \\ -2 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

is $3\sqrt{2}$

[5 marks]

Section C: The basics

7 A particle of mass 2 kg is raised a vertical distance of 8 m.

Find the work done against gravity.

Circle your answer.

4*g* Nm   16 Nm   4 J   16*g* J

[1 mark]

8 A car of mass 850 kg reduces speed from 25 m s^{-1} to 10 m s^{-1}.

Find the decrease in kinetic energy of the car.

[2 marks]

9 A particle of mass 8 kg which is initially at rest is pulled along a smooth, horizontal surface by a horizontal force of magnitude 20 N.

Using energy considerations, find the speed of the particle when it has been pulled along a distance of 15 m.

Give your answer as a simplified surd.

[4 marks]

10 A stone of mass 0.2 kg is dropped from a height, h m. The stone hits the ground with speed 45 m s^{-1}.

10 (a) Find h as a function of g, using the principle of conservation of energy.

[2 marks]

10 (b) State **two** assumptions made.

[1 mark]

Section D: A bit more thinking

11 **In this question use g = 9.8 m s^{-2}.**

A particle of mass 0.8 kg is attached to one end of a light elastic spring of natural length 2.4 m and modulus of elasticity 32 N.

The other end of the spring is attached to a fixed point, *O*, at the top of a rough plane inclined at an angle θ to the horizontal, where $\tan \theta = \frac{3}{4}$. The coefficient of friction between the particle and the plane is 0.25.

The particle is held at rest on the plane at a point 1 m from *O*, down the line of greatest slope of the plane. It is then released from rest and moves down the slope.

11 (a) Find the initial acceleration of the particle.

[4 marks]

11 (b) What is the significance of the spring being light?

[1 mark]

12 The resistance to motion of a car moving with speed v m s^{-1} is given by $(350 + 2v)$ N.

Given that the engine of the car is working at 10 kW, find the maximum speed of the car as it travels along a horizontal road.

[4 marks]

13 **In this question use g = 9.81 m s^{-2}.**

A boy and his skateboard have a combined mass of 60 kg. The boy descends a slope inclined at 15° to the horizontal, starting from rest.

At the bottom of the slope, the ground becomes horizontal for 18 m, before rising at 10° to the horizontal.

At the point where the boy has travelled 25 m up the slope, his speed is 5 m s^{-1}. He is subject to a constant resistance of magnitude 20 N throughout the motion.

By modelling the boy and his skateboard as a particle, and using energy considerations, find the distance the boy travels down the slope.

[5 marks]