AQA Realising potential

TOPIC TEST
MARK SCHEME

A-level
FURTHER MATHS

Further vectors & Work, Energy & Power
Mark scheme

Specification content coverage: F1, F3, F4, F6

Question	Solutions	Mark
1 (a)	Finds vector linking the two points, eg $\begin{pmatrix} -1 \\ -4 \\ 1 \end{pmatrix}$	1
	Finds correct line, eg $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -4 \\ 1 \end{pmatrix}$	1 (or equivalent)
	Total	**2**
1 (b)	Need to solve $\begin{aligned} 2-\lambda &= -6+2\mu \\ 3-4\lambda &= -2-\mu \\ \lambda-1 &= 4-\mu \end{aligned}$	1
	Solution $\lambda = 2$ and $\mu = 3$	1
	Intersection $(0,-5,1)$	1
	Total	**3**
2	$\cos(\theta) = \dfrac{14}{\sqrt{10}\sqrt{21}}$	1
	Hence $\theta = 15.0$ degrees (or equivalent)	1
	Total	**2**
3 (a)	$P(1+2t, 1-2t, -3-t)$	1
	Total	**1**
3 (b)	$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \begin{pmatrix} 1+2t \\ 1-2t \\ -3-t \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2t \\ -1-2t \\ -2-t \end{pmatrix}$	1
	Total	**1**

3 (c)	P is shortest distance from A when $\begin{pmatrix} 2t \\ 1-2t \\ -2-t \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} = 0$	1
	$4t+2+4t+2+t=0$	
	$t=-\frac{4}{9}$	1
	$\overrightarrow{AP} = \begin{pmatrix} -\frac{8}{9} \\ -\frac{1}{9} \\ -\frac{14}{9} \end{pmatrix}$	1
	$\lvert\overrightarrow{AP}\rvert = \sqrt{\left(\frac{8}{9}\right)^2+\left(\frac{1}{9}\right)^2+\left(\frac{14}{9}\right)^2} = \frac{\sqrt{261}}{9} = \frac{\sqrt{29}}{3} (=1.80)$	1
	Total	**4**
4 (a)	$\overrightarrow{OX} = \mathbf{i}+3\mathbf{j}-2\mathbf{k}+$ $\overrightarrow{YX} = (\mathbf{i}+3\mathbf{j}-2\mathbf{k})-(2\mathbf{i}+\mathbf{j}-\mathbf{k}) = (-\mathbf{i}+2\mathbf{j}-\mathbf{k})$	1
	$\cos OXY = \frac{-1+6+2}{\sqrt{14}\sqrt{6}} = \frac{7}{\sqrt{84}}$	1
	Total	**2**
4 (b)	Area = $\frac{1}{2}\times\sqrt{14}\times\sqrt{6}\times\sqrt{1-\left(\frac{7}{\sqrt{84}}\right)^2}$	1
	$=\frac{1}{2}\times\sqrt{84}\times\sqrt{\frac{35}{84}} = \frac{1}{2}\sqrt{35}\left(p=\frac{1}{2}, q=35\right)$	1
	Total	**2**
5 (a)	Forms equation $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 7+2\lambda \\ -1-\lambda \\ 2+3\lambda \end{pmatrix} = 0$	1
	Solves to find $\lambda = -\frac{3}{2}$	1
	Finds point $\left(4, \frac{1}{2}, -\frac{5}{2}\right)$	1
	Total	**3**
5 (b)	Finds distance $\frac{3\sqrt{10}}{2} \approx 4.74$ and concludes yes	1
	Total	**1**

6	If A is a point on $\mathbf{r}_1$, and B is a point on $\mathbf{r}_2$, $\overrightarrow{OA} = \begin{pmatrix} 3 \\ s \\ -1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 9+t \\ -2-2t \\ -1+t \end{pmatrix}$	1
	$\overrightarrow{AB} = \begin{pmatrix} 9+t \\ -2-2t \\ -1+t \end{pmatrix} - \begin{pmatrix} 3 \\ s \\ -1 \end{pmatrix} = \begin{pmatrix} 6+t \\ -2-2t-s \\ t \end{pmatrix}$	1
	$\overrightarrow{AB}$ is perpendicular to $\mathbf{r}_1$ so $\begin{pmatrix} 6+t \\ -2-2t-s \\ t \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0$ $-2-2t-s=0$	1
	$\overrightarrow{AB}$ is perpendicular to $\mathbf{r}_2$ so $\begin{pmatrix} 6+t \\ -2-2t-s \\ t \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = 0$ $6+t+4+4t+2s+t=0$ $10+6t+2s=0$ Solving gives $t=-3, s=4$	1
	$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}$ $\lvert\overrightarrow{AB}\rvert = 3\sqrt{2}$	1
	Total	**5**
7	$16g$ J	1
	Total	**1**
8	Initial KE = $\frac{1}{2} \times 850 \times 25^2$ = (265 625 J) Final KE = $\frac{1}{2} \times 850 \times 10^2$ = (42 500 J)	1
	Decrease in KE = 223 125 J	1
	Total	**2**
9	WD by the force = 20 × 15 = 300 J	1
	KE gained = $\frac{1}{2} \times 8 \times v^2 = 4v^2$	1
	$300 = 4v^2$	1
	$v = 5\sqrt{3}$ ms^{-1}	1

		Total	**4**
10 (a)	PE lost = $0.2gh$ KE gained = $\frac{1}{2} \times 0.2 \times 45^2$ $0.2gh = \frac{1}{2} \times 0.2 \times 45^2$ (= 202.5)		1
	$h = \dfrac{2025}{2g}$		1
		Total	**2**
10 (b)	Model stone as a particle, ignore air resistance		1
		Total	**1**
11 (a)	$T = \dfrac{32 \times 1.4}{2.4} = \dfrac{56}{3}$ N		1
	$R = 0.8g \cos\theta$ $= 0.8g \times \frac{4}{5}$ = 6.272 N		1
	$0.8g \sin\theta + T - 0.25R = 0.8a$ $7.84 \times \frac{3}{5} + \frac{56}{3} - 0.25 \times 6.272 = 0.8a$		1
	a = 27.2533… = 27 m s^{-2} (2 sf)		1
		Total	**4**
11 (b)	Negligible mass – does not contribute to weight/force pushing particle down slope. So a is less.		1
		Total	**1**
12	Driving force = $\dfrac{10000}{v}$		1
	$\dfrac{10000}{v} = 350 + 2v$		1
	$10000 = 350v + 2v^2$ $v^2 + 175v - 5000 = 0$ $(v - 25)(v + 200) = 0$ v = 25, –200		1 (both roots needed)
	Max speed = 25 m s^{-1}		1 (+ other root rejected)
		Total	**4**
13	KE gained = $\frac{1}{2} \times 60 \times 5^2$ (= 750 J)		1
	PE lost = $60g(x \sin 15 - 25 \sin 10)$ (= 152.3408… x – 2555.2329…)		1
	WD by resistance = $20 \times (x + 18 + 25)$ $= 20x + 860$		1
	$20x + 860 = 152.3408x - 2555.2329 - 750$ ($4165.2329 = 132.3408x$)		1
	Distance down the slope = 31.5 m (3 sf)		1
		Total	**5**

Grand Total: 50 Marks