# Check In – 2.03 Conditional Probability

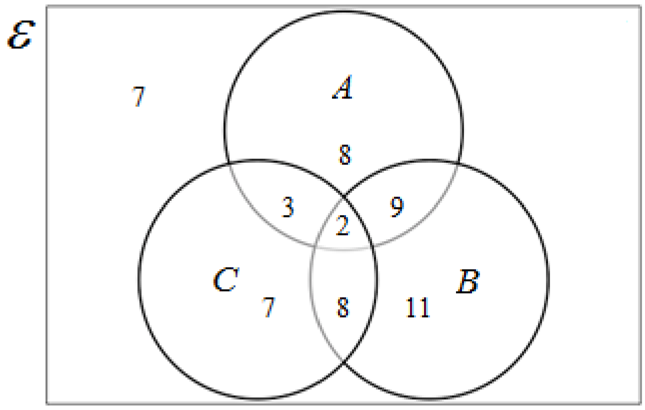
## Questions

1. The probability of two events $A$ and $B$ occurring and their intersection are $\mathrm{P}(A)=0.12$, $\mathrm{P}(B)=0.3$ and $\mathrm{P}(A\cap B)=0.035$.
   Calculate $\mathrm{P}(B\,|\,A)$.

2. Given that $\mathrm{P}(A)=\frac{4}{10}$, $\mathrm{P}(B)=\frac{14}{25}$ and $\mathrm{P}(A\,|\,B)=\frac{6}{7}$, calculate $\mathrm{P}(A\cap B)$.

3. Events $A$ and $B$ are mutually exclusive. The probability of event $B$ occurring and the union of $A$ and $B$ are $\mathrm{P}(B)=0.3$ and $\mathrm{P}(A\cup B)=0.5$.
   Find $\mathrm{P}(A)$.

4. The Venn diagram below shows the occurrences of events $A$, $B$ and $C$.




Find the following probabilities.

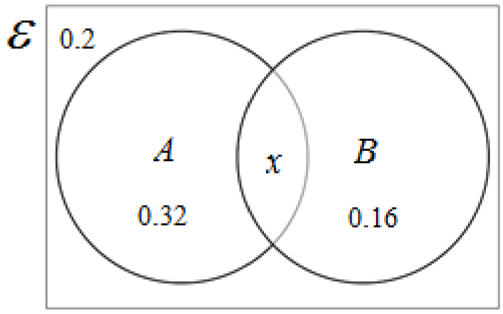
i) $\mathrm{P}(A)$
ii) $\mathrm{P}(A\cap B)$
iii) $\mathrm{P}(B\,|\,A)$
iv) $\mathrm{P}(C'\,|\,B)$

5. The table below shows some information about 200 students and their most frequent method of travel to school.

| | Walk | Not walk | Total |
|---|---|---|---|
| Male | 56 | 40 | 96 |
| Female | 72 | 32 | 104 |
| Total | 128 | 72 | 200 |

i) Find the probability that a randomly chosen student is female.
ii) Find the probability that a randomly chosen student is female and walks to school.
iii) Given the student is female what is the probability that they walk to school?

6. In the Venn diagram below the numbers represent probabilities.




Are events $A$ and $B$ independent? Explain your answer.

7. Two students are chosen at random from a class containing 6 boys and 7 girls.
Let $B$ represent the event that a boy is chosen at random and let $G$ represent the event that a girl is chosen at random.
Determine whether $\mathrm{P}(B|G)+\mathrm{P}(B'|G)=1$ is true or not.

8. There are $n$ red balls in a bag and $n-1$ blue balls in a bag. Eddie takes two balls out of the bag at random without replacement. The probability that two blue balls are chosen is $\frac{1}{5}$.

Show that $5\left(\frac{n-2}{2n-1}\right)=2$ and hence find the number of red balls.

9. Mike decides to buy 2 gerbils. He goes to a shop and the keeper reliably informs him that there are 7 male and 8 female gerbils to choose from. Mike choses two gerbils at random. Find the probability that both gerbils are female given they are the same sex.

10. A test for whether a patient has a particular condition has been trialled extensively by the pharmaceutical companies manufacturing the test. They find that 0.5% of the population have this particular medical condition. After testing, the outcomes are positive, negative or inconclusive.

It is found that if the patient has the condition then there is an 85% chance that they test positive, a 5% chance that they test negative or a 10% chance that the results are inconclusive. If the patient doesn't have the condition then there is a 90% chance that they test negative, a 5% chance that they test positive and a 5% chance that the tests are inconclusive.

If the results are inconclusive then the patient has to have a thorough examination. If they do not have the condition then the probability that a thorough examination determines they have the condition is 0.001%. If the patient does have the condition then the probability that a thorough examination detects this is 99%.

What is the probability that given they don't have the condition they test positive?

**Extension**

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## Worked solutions

1. $$\mathrm{P}(B|A)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(A)}$$
$$\mathrm{P}(B|A)=\frac{0.035}{0.12}=\frac{7}{24}$$

2. $$\mathrm{P}(A\cap B)=\mathrm{P}(B)\times\mathrm{P}(A|B)$$
$$\mathrm{P}(A\cap B)=\frac{14}{25}\times\frac{6}{7}=\frac{12}{25}$$

3. If two events $A$ and $B$ are mutually exclusive then their intersection vanishes and $\mathrm{P}(A)+\mathrm{P}(B)=\mathrm{P}(A\cup B)$.
In this case we have $\mathrm{P}(A)+0.3=0.5$ and hence $\mathrm{P}(A)=0.2$.

4. i) $$\mathrm{P}(A)=\frac{9+2+8+3}{7+7+8+11+9+2+8+3}=\frac{22}{55}=\frac{2}{5}$$

ii) $$\mathrm{P}(A\cap B)=\frac{9+2}{55}=\frac{11}{55}=\frac{1}{5}$$

iii) $$\mathrm{P}(B|A)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(A)}=\frac{\frac{1}{5}}{\frac{2}{5}}=\frac{1}{2}$$

iv) $$\mathrm{P}(C'|B)=\frac{\mathrm{P}(C'\cap B)}{\mathrm{P}(B)}=\frac{\frac{9+11}{55}}{\frac{30}{55}}=\frac{20}{30}=\frac{2}{3}$$

5. i) $$\mathrm{P}(\mathrm{Female})=\frac{104}{200}=\frac{13}{25}$$

ii) $$\mathrm{P}(\mathrm{Female}\cap\mathrm{Walk})=\frac{72}{200}=\frac{9}{25}$$

iii) $$\mathrm{P}(\mathrm{Walk}|\mathrm{Female})=\frac{\mathrm{P}(\mathrm{Female}\cap\mathrm{Walk})}{\mathrm{P}(\mathrm{Female})}=\frac{72}{104}=\frac{9}{13}$$

6. Two events $A$ and $B$ are independent if $\mathrm{P}(A)\times\mathrm{P}(B)=\mathrm{P}(A\cap B)$.
$\mathrm{P}(A\cap B)=x$
$x+0.2+0.32+0.16=1$
$\Rightarrow x+0.68=1$
$\Rightarrow x=0.32$
In this case we have $0.64\times0.48=0.3072\neq\mathrm{P}(A\cap B)$ hence they are not independent.

7. Finding the individual probabilities:

$$\mathrm{P}(B \cap G) = \left(\frac{6}{13} \times \frac{7}{12}\right) = \frac{7}{26}$$

$$\mathrm{P}(B \mid G) = \frac{\mathrm{P}(B \cap G)}{\mathrm{P}(G)} = \frac{\frac{7}{26}}{\frac{7}{13}} = \frac{1}{2}$$

$$\mathrm{P}(B' \cap G) = P(G \cap G) = \left(\frac{7}{13} \times \frac{6}{12}\right) = \frac{7}{26}$$

$$\mathrm{P}(B' \mid G) = \frac{\mathrm{P}(B' \cap G)}{\mathrm{P}(G)} = \frac{\frac{7}{26}}{\frac{7}{13}} = \frac{1}{2}$$

Therefore we can verify that $\mathrm{P}(B \mid G) + P(B' \mid G) = \frac{1}{2} + \frac{1}{2} = 1$.

8. $$\mathrm{P}(\text{Blue and Blue}) = \frac{n-1}{2n-1} \times \frac{n-2}{2n-2} = \frac{1}{5}$$

$$= \frac{n-1}{2n-1} \times \frac{n-2}{2(n-1)} = \frac{1}{5}$$

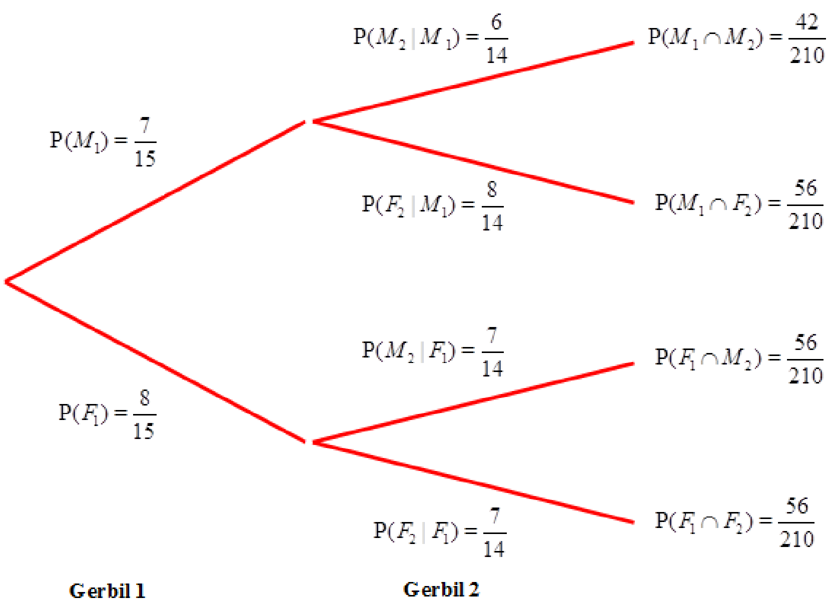
$$= \frac{n-2}{2(2n-1)} = \frac{1}{5}$$

and hence $\quad 5\left(\frac{n-2}{2n-1}\right) = 2$.

The number of red balls, $n$, by rearranging and solving gives:

$$5(n-2) = 2(2n-1)$$
$$5n - 10 = 4n - 2$$
$$n = 8$$

9.

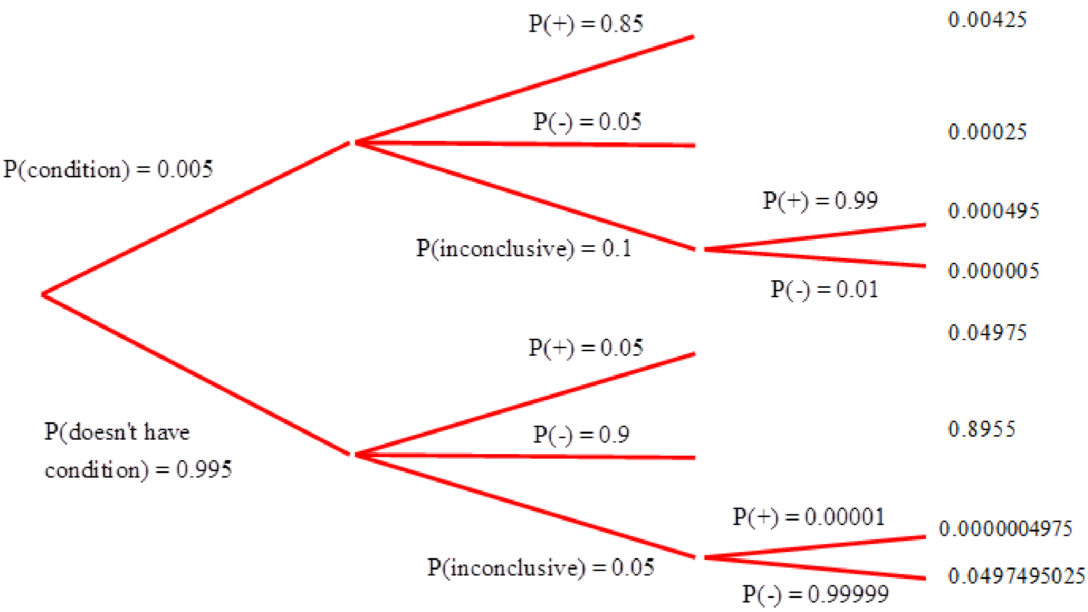


$$P(\text{Same sex}) = P(\text{Both male}) + P(\text{Both female}) = \frac{42}{210} + \frac{56}{210} = \frac{98}{210} = \frac{7}{15}$$

$$P(\text{Both female}|\text{Same sex}) = \frac{P(\text{Both female} \cap \text{Same sex})}{P(\text{Same sex})} = \frac{\frac{56}{210}}{\frac{98}{210}} = \frac{56}{98} = \frac{4}{7}$$

10.




$$P(\text{Test positive}|\text{Don't have condition}) = \frac{P(\text{Test positive} \cap \text{Don't have condition})}{P(\text{Don't have condition})}$$

$$= \frac{0.04975 + 0.0000004975}{0.995} = 0.0500005$$

## Extension

'Squash' by nrich.maths.org
https://nrich.maths.org/342/solution