Section Check In – 2.05 Statistical Hypothesis Testing

Questions

1. A supermarket typically has 60% of inhabitants of a town who shop there. The manager wants to see if the new advertising campaign has increased the number of customers.

   State the null and alternative hypothesis for this test.

2. A random sample of size n is taken from a binomial distribution. If $n = 20,\ x = 18,$ $\mathrm{H}_0 : p = 0.7,\ \mathrm{H}_1 : p > 0.7$ carry out a hypothesis test given the null and alternative hypotheses. Test at the 3% significance level.

3.* A random sample of size n is taken from a normally distributed population with a given standard deviation. If $\sigma = 15,\ n = 10,\ \bar{x} = 140,\ \mathrm{H}_0 : \mu = 150,\ \mathrm{H}_1 : \mu \neq 150$ carry out a hypothesis test given the null and alternative hypotheses. Test at the 1% significance level.

4.* A random sample of size n is taken from a normally distributed population with a given standard deviation. If $\sigma = 40,\ n = 50,\ \bar{x} = 238,\ \mathrm{H}_0 : \mu = 230,\ \mathrm{H}_1 : \mu > 230$ carry out a hypothesis test given the null and alternative hypotheses. Test at the 10% significance level.

5. A television programme called “Cook off” changes three of its presenters. The programme was viewed by 20% of a population of 50 million. The producers want to determine if this change has affected viewing figures. From a random sample of 100 from the population, 28 said they watched the programme.

   Carry out a suitable hypothesis test at the 5% significance level (stating the null and alternative hypotheses) to determine whether there is sufficient evidence to suggest viewing figures have changed.

6.* The lifetimes of a particular type of light bulb has been normally distributed with mean 4500 hours and standard deviation 500 hours. A quality control manager suspects that changes in production methods have led to lower mean lifetime but no change in the standard deviation. A random sample of 8 light bulbs will be used to test the quality control manager’s suspicion. The mean lifetime for the sample will be used as the test statistic.

   (i) State suitable null and alternative hypotheses for the test.
   (ii) Find the critical region for the test at the 3% significance level.

7. Harry is taking a multiple choice test with 40 questions. Each question has 4 answers to choose from. Harry gets 2 questions right; he says he guessed them all. Harry’s teacher says that his mark is worse than someone who is just guessing. Carry out a hypothesis test with a 5% level of significance to see if there is evidence to support the teacher’s claim.

8.* An athlete's times for running 400 m are normally distributed with mean 55 seconds and standard deviation 3 seconds. A manufacturer of a new trainer claims that it improves running times. The athlete asks to trial it to see if it improves his times. He times a random sample of 10 runs of 400 m each and his mean time was 53 seconds. Carry out a suitable hypothesis test (stating the null and alternative hypotheses) to investigate the manufacturer's claim at the 3% significance level. Assume that there is no change in the standard deviation.

9. Coeliacs are people who must avoid eating gluten. A society monitoring restaurants that offer gluten free menus has established that around 20% of restaurants offer a separate gluten free menu. A local newspaper wants to promote the availability of gluten free dishes and claims that there are more restaurants than this in the area. Of 20 restaurants contacted, 10 said that they do offer a separate menu.

   (i) Conduct a suitable hypothesis test with a 5% significance level to see whether there is evidence to support the newspaper's claim.
   (ii) Determine what the p-value would be for the test.

10.* Average attendances at a football club last season were normally distributed with mean $72\,000$ and standard deviation $1\,500$. Ticket prices were increased this season and the mean attendance from the first 6 matches has been $70\,000$. Carry out a hypothesis test with a 5% level of significance to determine if the season ticket price has caused attendances to drop.

Extension

A football pundit has suggested that the more revenue a football team has, the more points it will win in a season. In 2015/2016, the English Premier League teams had the following points and revenue in € millions.

Team	**Revenue €m**	**Points**
Manchester United	689	66
Manchester City	525	66
Arsenal	469	71
Chelsea	447	50
Liverpool	404	60
Tottenham Hotspur	280	70
West Ham United	192	62
Leicester City	172	81
Newcastle United	168	37
Southampton	163	63
Everton	163	47
Sunderland	144	39

Source: Deloitte Money Football League
https://www2.deloitte.com/uk/en/pages/sports-business-group/articles/deloitte-football-money-league.html
Premier League Table
https://www.premierleague.com/tables

(i) Show that $r = 0.3263$ (4 dp) given that $r = \dfrac{\sum xy - \dfrac{\sum x \sum y}{n}}{\sqrt{\left(\sum x^2 - \dfrac{(\sum x)^2}{n}\right)\left(\sum y^2 - \dfrac{(\sum y)^2}{n}\right)}}$.

(ii) State the null and alternative hypotheses.

(iii) Carry out a hypothesis test at the 5% level to determine whether there is any connection between revenue and points won. State your conclusion in context.

(iv) Do your findings show that there is definitely no relationship?

Worked solutions

1. $\mathrm{H}_0 : p = 0.6,\ \mathrm{H}_1 : p > 0.6$, where p is the proportion of inhabitants that shop at the supermarket.

2. $X \sim \mathrm{B}(20, p)$
$\mathrm{H}_0 : p = 0.7,\ \mathrm{H}_1 : p > 0.7$
If H_0 is true, $\mathrm{P}(X \geq 18) = 0.03548$
This is a one-tail test, $0.03548 > 0.03$
Therefore, as this is not in the critical region we do not reject H_0 which means there is insufficient evidence against the null hypothesis.

3. $X \sim N(\mu, 15^2)$
$\mathrm{H}_0 : \mu = 150,\ \mathrm{H}_1 : \mu \neq 150$
If H_0 is true, then the sample mean, $\bar{X} \sim \mathrm{N}\left(150,\ \dfrac{15^2}{10}\right)$
$\mathrm{P}(\bar{X} \leq 140) = 0.0175$
This is a two-tail test, $0.0175 > 0.005$
Therefore, as this is not in the critical region we do not reject H_0 which means there is insufficient evidence against the null hypothesis.

4. $X \sim N(\mu, 40^2)$
$\mathrm{H}_0 : \mu = 230,\ \mathrm{H}_1 : \mu > 230$
If H_0 is true, then the sample mean, $\bar{X} \sim \mathrm{N}\left(230,\ \dfrac{40^2}{50}\right)$
$\mathrm{P}(\bar{X} \geq 238) = 0.07865$
This is a one-tail test, $0.07865 < 0.1$
Therefore, as this is in the critical region we reject H_0 which means there is sufficient evidence against the null hypothesis.

5. Let X be the number of viewers from the sample of 100 watching the “Cook off” and p the proportion of the population who watch the “Cook off”.
$X \sim \mathrm{B}(100, p)$
$\mathrm{H}_0 : p = 0.2,\ \mathrm{H}_1 : p \neq 0.2$
Under H_0, $X \sim \mathrm{B}(100, 0.2)$
$\mathrm{P}(X \geq 28) = 1 - \mathrm{P}(X \leq 27)$
$\mathrm{P}(X \geq 28) = 1 - 0.9658 = 0.0342$
This is a two-tail test, $0.0342 > 0.025$
Therefore, as this is not in the critical region we do not reject H_0 which means there is insufficient evidence against the null hypothesis that the new presenters have affected viewing figures.

6. (i) $H_0 : \mu = 4500,\ H_1 : \mu < 4500$ where μ is the population mean lifetime for the light bulbs.

(ii) Let X be the lifetime of a light bulb.

$X \sim N(\mu, 500^2)$

If H_0 is true, then the sample mean, $\bar{X} \sim N\left(4500,\ \frac{500^2}{8}\right)$

This is a one-tail test. The critical value, k, is such that $P(\bar{X} < k) = 0.03$

Using the inverse Normal, $k = 4168$ (4sf)

The critical region is $\bar{x} < 4168$

7. Let X be the number of questions that someone gets correct and p represents the probability of getting a question correct.

$X \sim B(40, p)$

$H_0 : p = 0.25,\ H_1 : p < 0.25$

Under H_0, $X \sim B(40, 0.25)$.

$P(X \le 2) = 0.001$

This is a one-tail test, $0.001 < 0.05$ so there is evidence that Harry is doing worse than someone who is just guessing.

8. Let T be the times in seconds achieved by an athlete running 400 m.

$T \sim N(55, 3^2)$

$H_0 : \mu = 55,\ H_1 : \mu < 55$

If H_0 is true, then the sample mean, $\bar{T} \sim N\left(55,\ \frac{3^2}{10}\right)$

$P(\bar{T} \le 53) = 0.0175$

This is a one-tail test, $0.0175 < 0.03$

Therefore, as this is in the critical region we reject H_0 which means there is sufficient evidence against the null hypothesis suggesting that the new trainers do improve the running times of an athlete.

9. a) Let G be the number of restaurants offering gluten free menus and p represents the proportion of restaurants that offer a gluten free menu.

$G \sim B(20, p)$

$H_0 : p = 0.2,\ H_1 : p > 0.2$

Under H_0, $X \sim B(100, 0.2)$

$P(G \ge 10) = 1 - P(X \le 9)$

$P(G \ge 10) = 1 - 0.997 = 0.003$

This is a one-tail test, $0.003 < 0.05$

Therefore, as this is in the critical region we reject H_0 which means there is sufficient evidence against the null hypothesis suggesting there is evidence to support the local paper's claim that there are more than 20% of restaurants in the area offering a separate gluten free menu.

b) This is a one-tail test. The critical value, g, is such that $P(G > g) = 0.05$

$\mathrm{P}(G \geq 7) = 1 - \mathrm{P}(G \leq 6) = 1 - 0.9133 = 0.0867$

$\mathrm{P}(G \geq 8) = 1 - \mathrm{P}(G \leq 7) = 1 - 0.9679 = 0.0321$

Therefore, the test statistic is to reject H_0 if $G \geq 8$ and the p-value is 0.0321.

10. Let X be the attendance in thousands at a football ground.

$X \sim N(72, 1.5^2)$

$\mathrm{H}_0 : \mu = 72,\ \mathrm{H}_1 : \mu < 72$

If H_0 is true, then the sample mean, $\bar{X} \sim \mathrm{N}\left(72,\ \dfrac{1.5^2}{6}\right)$

$\mathrm{P}(\bar{X} \leq 70) = 0.0005$

This is a one-tail test, $0.0005 < 0.05$

Therefore, as this is in the critical region we reject H_0 which means there is sufficient evidence against the null hypothesis suggesting that the increase in ticket prices may have caused the attendances to decrease this season.

Extension

(i) Using a calculator $r = 0.3263032535$

(ii) $\mathrm{H}_0 : p = 0,\ \mathrm{H}_1 : p \neq 0$ where p is the correlation coefficient for all football team revenues and their points totals.

(iii) The critical value for a two-tail test at the 5% significance level for $n = 12$ is 0.5760.
As $0.3263 < 0.5760$ we do not reject H_0 as there is insufficient evidence against the null hypothesis. This suggests that there is not a relationship between revenue and points won in the Premier League.

(iv) As the product moment correlation coefficient tests for how close the points lie to a straight line it may be that there is a non-linear relationship between the two variables. It may be more appropriate to look at the ranking positions of teams and their revenue. As the sample is small it may also mean that there is an extreme value (e.g. Leicester City) which has distorted the data so a greater sample would make the findings more reliable.

