Section Check In – 2.03 Probability

Questions

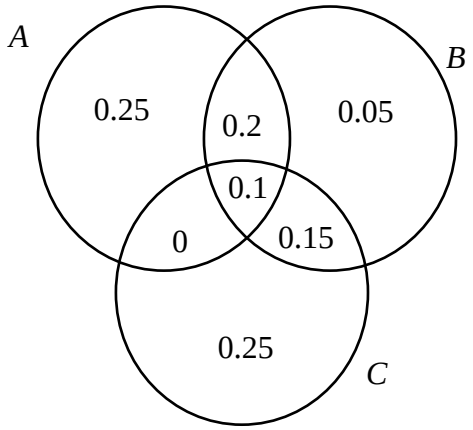
1. The random variable X has the probability distribution shown in this table.

x	0	1	2	3
$\mathrm{P}(X = x)$	0.1	0.2	0.3	0.4

A is the event that $X > 2$. Find $\mathrm{P}(A)$.

2. A and B are independent events. $\mathrm{P}(A) = 0.35$ and $\mathrm{P}(B) = 0.04$.
Calculate $\mathrm{P}(A \text{ and } B)$.

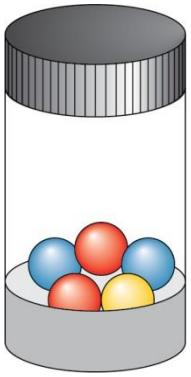
3.* This Venn diagram shows the probabilities associated with events A, B and C.

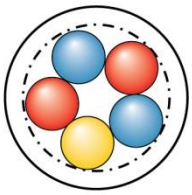


Calculate $\mathrm{P}(A / (B \cap C))$.

4.* $\mathrm{P}(A) = \frac{1}{2}$ $\mathrm{P}(B) = \frac{3}{5}$ $\mathrm{P}(A \cup B) = \frac{17}{20}$
Calculate $\mathrm{P}(A \cap B)$.

5. A tube contains 5 balls. Two of the balls are red, two are blue and one is yellow.
Mia shakes the tube and the balls fall into a ring.





Use a sample space diagram to explain why the probability that the two blue balls touch is $\frac{1}{2}$.

6.* The following table classifies members of the UK parliament (MPs) according to their political party and gender.

	Labour	Conservative	Other	**Total**
Male	129	261	65	**455**
Female	101	69	25	**195**
Total	**230**	**330**	**90**	**650**

http://www.ukpolitical.info/female-members-of-parliament.htm April 2017

Explain why this data suggests that gender and party affiliation among MPs are not independent.

7. In a school class, half the pupils represent the school at a winter sport, one third represent the school at a summer sport and one tenth do both.
A student is chosen at random from this class. Find the probability that they represent the school at sport.

8.* The probability that a person is left-handed is 0.08. 91% of left-handers and 1% of right-handers are left-footed. What is the probability that a person chosen at random catches a ball with one hand but kicks it with the opposite foot?

9. Idris works five days a week, Monday to Friday.
The chance that he gets up late on any working day is 0.3.
Last week, Idris got up late **exactly once** during the working week. What percentage of working weeks would you expect this to happen?

10.* Ava has three cards. One card has both sides red, one has both sides black, and one has one red side and one black side.
Ava holds up a random card with a random side facing you. The side you see is red.
What is the probability that the other side of the same card is red?

Extension

In a game, three counters marked 1, 4 and 3 are placed in a bag. Tom draws a counter from the bag at random, records the number and then replaces the counter in the bag. He repeats this until the numbers he has drawn sum to 5 or more. Tom wins if the numbers have a sum of exactly 5. If the sum is more than 5, he loses.

List all the possible winning combinations of numbers that Tom could draw, and all the possible losing ones. For example, $(1, 1, 3)$ would win but $(1, 1, 1, 3)$ would lose.

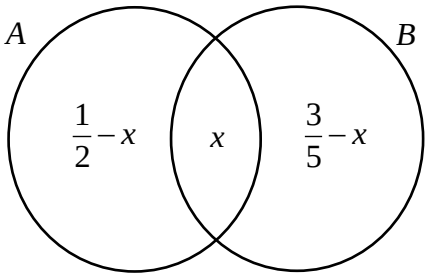
Worked solutions

1. $X > 2 \Rightarrow x = 3$ and $\mathrm{P}(x = 3) = 0.4$

2. Because A and B are independent, $\mathrm{P}(A \text{ and } B) = \mathrm{P}(A) \times \mathrm{P}(B)$
$= 0.35 \times 0.04 = 0.014$

3. This is the probability of A given (B and C).
The probability of (B and C) is $0.1 + 0.15 = 0.25$. The proportion of (B and C) that is also A is then $\frac{0.1}{0.25} = 0.4$, or $\frac{\mathrm{P}(A \cap B \cap C)}{\mathrm{P}(B \cap C)}$

4. This can be answered using a Venn diagram:

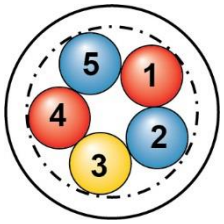


So $\frac{1}{2} - x + x + \frac{3}{5} - x = \frac{17}{20} \Rightarrow x = \frac{1}{4}$

$\mathrm{P}(A \cap B) = x$ so $\mathrm{P}(A \cap B) = \frac{1}{4}$

Or, without a diagram, $\mathrm{P}(A \cup B) = \mathrm{P}(A) + P(B) - \mathrm{P}(A \cap B) \Rightarrow \mathrm{P}(A \cap B) = \frac{1}{2} + \frac{3}{5} - \frac{17}{20} = \frac{1}{4}$

5. If we label the positions 1 to 5 then there are 10 possible positions for the two blue balls:



(1, 2) (1, 3) (1, 4) (1, 5) (2, 3) (2, 4) (2, 5) (3, 4) (3, 5) (4, 5)

Of these, the blue balls are touching in 5 possible positions: (1, 2) (1, 5) (2, 3) (3, 4) (4, 5)

So the probability is $\frac{5}{10} = \frac{1}{2}$

6. For independence, the probability that a person is male (or female) should be the same regardless of their party affiliation. However,

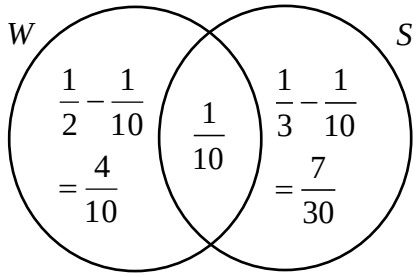
P(Male/Labour) $= \frac{129}{230} = 0.561$ and P(Male/Conservative) $= \frac{261}{330} = 0.791$

So gender and party affiliation are dependent.

Alternatively, P(Labour/Male) $= \frac{129}{455} = 0.284$ and P(Labour/Female) $= \frac{101}{195} = 0.518$

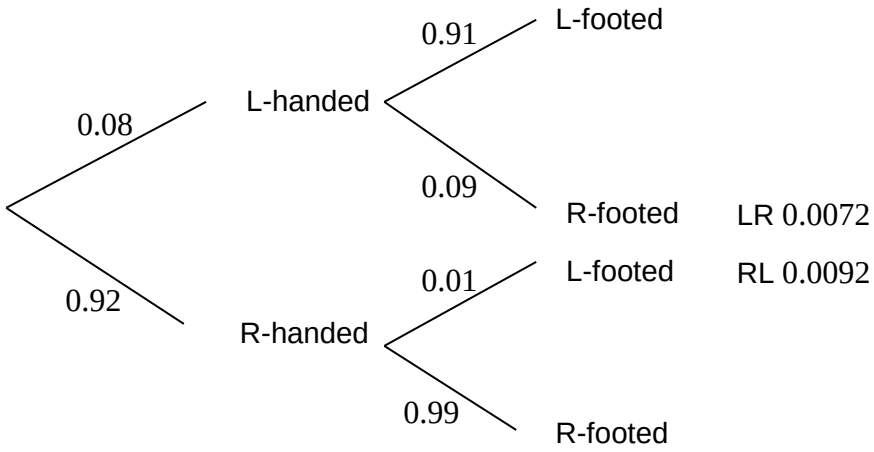
So again, gender and party affiliation are shown to be dependent.

7.



So the probability that someone chosen at random represents the school at sport is $\frac{4}{10} + \frac{1}{10} + \frac{7}{30} = \frac{11}{15}$

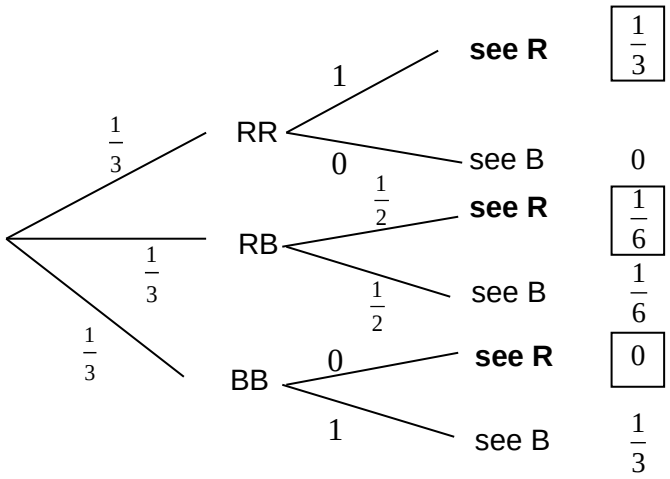
8.



So the required probability is 0.0164, or 1.64% of the population.

9. The probability that Idris gets up late exactly once in a working week is $0.3 \times 0.7 \times 0.7 \times 0.7 \times 0.7 \times 5 = 0.36015$
So we would expect this to happen in about 36% of working weeks.

10.



We want P(card was RR given that you see R) $= \dfrac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{6}+0} = \dfrac{\frac{1}{3}}{\frac{1}{2}} = \dfrac{2}{3}$

Extension

This activity is about being systematic and logical in constructing a sample space.

Ways of winning (1, 1, 1, 1, 1) (1, 1, 3) (1, 3, 1) (3, 1, 1) (1, 4) (4, 1)

Ways of losing (1, 1, 1, 1, 3) (1, 1, 1, 1, 4) (1, 1, 1, 3) (1, 1, 1, 4) (1, 1, 4) (1, 3, 3)
(3, 1, 3) (3, 1, 4) (1, 3, 4) (3, 4) (4, 3) (3, 3) (4, 4)

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