Section Check In – 2.04 Statistical Distributions

Questions

1. The probability distribution of a discrete random variable X is given by
$P(X = x) = \frac{kx}{4}$ for $x = 1, 2, 3, 4$
Find the value of k and tabulate the probability distribution of X.

2. If $X \sim B(14, 0.3)$ find $P(X = 2)$.

3.* If $X \sim N(150, 16)$ find $P(X \geq 140)$.

4.* $X \sim N(100, \sigma^2)$ and $P(X < 115) = 0.7$
Find σ.

5. An optician has ten appointments a day. The number of these appointments of which someone is classed as being short-sighted is denoted by S. The variable S is modelled by the distribution $S \sim B(10, 0.35)$. Show that according to this model, the optician is more likely to see four people who are short-sighted in one day than two people.

6.* Kathy is the caterer for a wedding of 180 guests. She decides to produce a buffet with 2 main courses (a meat option and a vegetarian option). From previous experience, she knows 55% of people in the area will opt for the meat option.

   (i) Describe a distribution that could be used to model this scenario. Explain, in context, the assumptions used.

   Kathy decides to use the normal distribution as an approximation to the binomial distribution so that she can more easily calculate how likely it is that she will have enough meat meals available.

   (ii) What would the parameters be for this distribution?

7. John organises a running race around a park. He assumes that the probability of each person entering the race being a male is 0.7.

   (i) Calculate the probability that the first 20 people to enter are all males.

   (ii) Calculate the probability that the first 6 people to enter the race will comprise three males and three females.

specification, therefore please use the information on the latest specification at all times. If you do notice a discrepancy please contact us on the following email address: resources.feedback@ocr.org.uk

8.* Greg is obsessed with solar lights. He purchases some from an online supplier, Sahara. The package states that each light shines for a time that is Normally distributed with mean of 5 hours and a standard deviation of 1.5.

(i) Calculate the probability that a randomly chosen solar light only shines for 4 hours or less.

(ii) Between what lengths of time would you expect 95% of all the solar lights to shine for?

(iii) What else would you suggest that the manufacturer adds to the packaging regarding the amount of time the solar lights will be on for?

9.* Michelle drives to work and on any random day the probability that she is late is 0.4. Her boss Julie says that if she is late more than two times over a one month period (assume 20 working days) she will have a formal written warning.

(i) State what distribution could be used to model this scenario. (You should give any parameters and state any assumptions.)

(ii) Calculate the probability of Michelle getting a formal written warning this month.

10.* A manufacturer of bags of sweets models the weight of each bag W by the distribution $W \sim N(\mu, 4)$. If the manufacturer wants to be 95% certain he has at least 50g in each bag, what would the value of μ be?

Extension

A sports analyst looks at the times that Alastair achieves over a five-year period in triathlon. He puts the times into statistical software which gives the following information:

Min = 200 min
Lower Quartile = 210 min
Median = 212 min
Upper Quartile = 225 min
Max = 250 min

He decides to model the times, T, as Normally distributed $T \sim N(\mu, \sigma^2)$.

(i) Using the Upper and Lower Quartile times, find μ and σ.

(ii) Using this distribution, find the probability that he will achieve a personal best in his next race.

(iii) Comment on the suitability of this model.

Worked solutions

1. $\frac{k}{4}+\frac{2k}{4}+\frac{3k}{4}+\frac{4k}{4}=1\Rightarrow\frac{10k}{4}=1\Rightarrow k=0.4$

X	1	2	3	4
$P(X=x)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

2. $P(X=2)=\binom{14}{2}\times0.3^2\times0.7^{12}=0.113\ (3\text{sf})$ or by using calculator probability functions (**BC**).

3. $\mu=150,\quad\sigma=4,\quad P(X\geq140)=0.99379$ **BC**

4. $P(X<115)=0.7$
   Standardising
   $P\left(Z<\frac{115-100}{\sigma}\right)=0.7$
   Using the inverse of the standard Normal distribution,
   $\frac{15}{\sigma}=0.52440051$
   $\sigma=28.6\ (1\text{ d.p.})$

5. $P(X=4)=\binom{10}{4}\times0.35^4\times0.65^6=0.238\ (3\text{sf})$
   $P(X=2)=\binom{10}{2}\times0.35^2\times0.65^8=0.176\ (3\text{sf})$

6. (i) $X\sim B(180,0.55)$
   Assumptions: The probability of someone choosing a meat dish remains constant; the probability for the wedding guests is the same as for the local area (0.55); whether someone chooses a meat dish is independent of anyone else's choice; each person either chooses a meat dish or the vegetarian dish.

   (ii) $\mu=np=180\times0.55=99$
   $\sigma^2=npq=180\times0.55\times0.45=44.55$
   $X\ \square\ N(99,44.55)$

7. (i) $0.7^{20}=0.000798\ (3\text{sf})$

   (ii) Let X be the number of males in the first 6 to enter the race.
   $X\sim B(6,0.7)\Rightarrow P(X=3)=\binom{6}{3}\times0.7^3\times0.3^3=0.18522$
   If 3 are male then the other 3 are female.

8. (i) $X \sim N(5,1.5^2) \Rightarrow P(X \leq 4) = 0.25249$ **BC**

(ii) 95% of values lie within two standard deviations of the mean. Therefore, you would expect the number of hours that 95% of the solar lights are on for are between 2 and 8 hours.

(iii) The position of the solar lights, weather conditions or the time of year may mean they are not fully charged.

9. (i) This problem can be modelled using a binomial distribution using $n = 20$ and probability of being late $= 0.4$. Assume that whether Michelle is late one day is independent of any other day and the probability of being late each day remains constant.

(ii) $B \sim (20, 0.4)$
$P(X > 2) = 0.996 \text{ (3sf)}$ **BC**

10. $X \sim N(\mu, 4)$

$P(X \geq 50) = 0.95 \Rightarrow P(Z \geq \frac{50-\mu}{\sqrt{4}}) = 0.95$

$P(Z < \frac{\mu-50}{\sqrt{4}}) = 0.95$

Using the inverse of the standard Normal distribution,

$(\frac{\mu-50}{\sqrt{4}}) = (1.645) \Rightarrow \mu = 53.3g \text{ (3sf)}$

Extension

(i) $P(T < 210) = 0.25, P(T < 225) = 0.75$

Standardise to achieve two simultaneous equations

$\mu = 0.75\sigma + 210, \ \mu = 225 - 0.75\sigma$

$\mu = 217.5, \ \sigma = 10$

(ii) $P(T < 200) = 0.0401$ **BC**

(iii) The median is closer to the lower quartile so this suggests the data is not normally distributed. You would need to consider the conditions on the day / course as well.

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