Realising potential

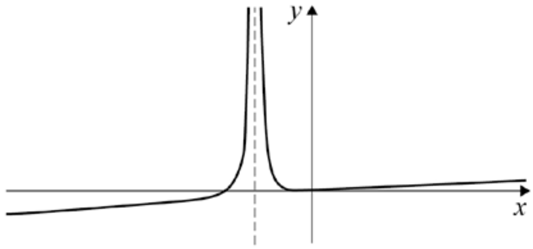
TOPIC TEST
MARK SCHEME

AS and A-level MATHS

Differentiation I

Mark scheme

Specification content coverage: G1, G2

Question	Solutions	Mark
1		1
	Total	**1**
2 (a)	$(2+h)^3 = h^3 + 6h^2 + 12h + 8$	1 $p = 12$ 1 $q = 8$
	Total	**2**
2 (b)	When $x = 2+h$, $y = h^3 + 6h^2 + 12h + 8 - 5(2+h)$ $= h^3 + 6h^2 + 12h + 8 - 10 - 5h$ $= h^3 + 6h^2 + 7h - 2$ Gradient of chord *PQ* $= \dfrac{(h^3 + 6h^2 + 7h - 2) - (-2)}{(2+h) - (2)}$ $= \dfrac{h^3 + 6h^2 + 7h}{h} = h^2 + 6h + 7$ As $h \to 0$, $h^2 + 6h + 7 \to 7$. Therefore gradient of tangent is 7.	1 Correct y value at Q 1 Finding gradient of chord *PQ* 1 Correct conclusion, including limiting process
	Total	**3**

3 (a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4x^3 - 9x^2 + 2$	1 one term correct 1 all correct
	Total	**2**
3 (b)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 12x^2 - 18x$	1
	Total	**1**
4	$\mathrm{f}(x) = -4x^3 + 20x^2 - 13x - 12$ $\mathrm{f}'(x) = -12x^2 + 40x - 13$	1 1 one term correct 1 all correct
	Total	**3**
5 (a)	$x^{\frac{5}{2}}$	1
	Total	**1**
5 (b)	$y = x^{\frac{5}{2}} - 3x^{-1}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5}{2}x^{\frac{3}{2}} + 3x^{-2}$	2 (one mark for each term differentiated correctly)
	Total	**2**
6	In Step 2, $(-4+h)^2$ should equal $16 - 8h + h^2$ In Step 4, he should consider the limit as $h \to 0$, rather than just letting $h = 0$.	1 1
	Total	**2**
7 (a)	$\frac{\mathrm{d}h}{\mathrm{d}x} = x^{\frac{1}{3}} - 0.6x$ When x = 1, $\frac{\mathrm{d}h}{\mathrm{d}x} = 0.4$ At this point, the gradient of the hill is such that for each (kilo)metre travelled horizontally, the height of the hill will increase by 0.4 (kilo)metres.	1 1 1 referencing rate of change of height
	Total	**3**

7 (b)	$\frac{\mathrm{d}^2h}{\mathrm{d}x^2}=\frac{1}{3}x^{-\frac{2}{3}}-0.6$ When x = 1, $\frac{\mathrm{d}^2h}{\mathrm{d}x^2}=-0.267$ (3s.f.) At this point, the gradient of the hill is decreasing at a rate of 0.267 for each (kilo)metre travelled horizontally.	1 1 referencing rate of change of gradient
	Total	**2**
7 (c)	In reality, a hill is not smooth so model won't give an accurate measurement of true height. OR Model predicts that height will eventually become negative, which can't happen in reality.	1
	Total	**1**
8	$\mathrm{f}(x)=(5+2x^{\frac{1}{2}})^3=5^3+3(5^2)(2x^{\frac{1}{2}})+3(5)(4x)+(8x^{\frac{3}{2}})$ $=125+150x^{\frac{1}{2}}+60x+8x^{\frac{3}{2}}$ $\mathrm{f}'(x)=75x^{-\frac{1}{2}}+60+12x^{\frac{1}{2}}$	1 Use of binomial 1 Correct expansion 1 One term correct 1 All correct
	Total	**4**
9	$y=3x^3+4x^{-1}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=9x^2-4x^{-2}$ Putting $\frac{\mathrm{d}y}{\mathrm{d}x}=-35$ $9x^2-4x^{-2}=-35$ Rearranging to form polynomial equation $9x^4+35x^2-4=0$ Solutions $x=\frac{1}{3},\ x=-\frac{1}{3}$ Find coordinates $\left(\frac{1}{3},\frac{109}{9}\right)$ and $\left(-\frac{1}{3},-\frac{109}{9}\right)$	 1 1 (Method) 1 (Method) 1 1
	Total	**5**
	TOTAL	**32**