AQA Realising potential

TOPIC TEST
MARK SCHEME

AS and A-level MATHS

Differentiation II

Mark scheme

Specification content coverage: G3

Question	Solutions	Mark
1 (a)	$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{6}x^2$ When $x = 9$, $\frac{dy}{dx} = \frac{41}{3}$	2 1
	Total	**3**
1 (b)	Gradient of normal is $-\frac{3}{41}$ Equation of normal is $y - \frac{87}{2} = -\frac{3}{41}(x-9)$	1 1
	Total	**2**
2 (a)	$\frac{dy}{dx} = 24x + 3x^{-2}$	2
	Total	**2**
2 (b)	Stationary point when $\frac{dy}{dx} = 0$ $24x + \frac{3}{x^2} = 0 \Rightarrow 24x^3 + 3 = 0 \Rightarrow x^3 = -\frac{1}{8}$ $x = -\frac{1}{2}$	1 Putting $\frac{dy}{dx} = 0$ and attempting to rearrange for x^3 1 Finding x
	Total	**2**

2 (c)	$\frac{d^2y}{dx^2} = 24 - 6x^{-3}$ When $x = -\frac{1}{2}$, $\frac{d^2y}{dx^2} = 72 > 0$ Therefore stationary point is minimum.	1 1 (must include reference to sign of second derivative)
	Total	**2**
3 (a)	$f'(x) = 6x^2 + 4x - 42$ Decreasing means $f'(x) < 0$ $6x^2 + 4x - 42 < 0 \Rightarrow 3x^2 + 2x - 21 < 0$	1 1
	Total	**2**
3 (b)	$-3 < x < \frac{7}{3}$	1 Correct critical values used in an inequality 1 Correct inequality
	Total	**2**
4 (a)	$\frac{dP}{dt} = 2.01t^2 - 26.64t + 70$ Stationary points occur at $t = 3.61$ and $t = 9.64$ $\frac{d^2P}{dt^2} = 4.02t - 26.64$ When $t = 3.61$, $\frac{d^2P}{dt^2} = -12.1 < 0$ therefore maximum When $t = 9.64$, $\frac{d^2P}{dt^2} = 12.1 > 0$ therefore minimum Minimum cost when $t = 9.64$.	1 1 1 1
	Total	**4**
4 (b)	$t = 9.64$ corresponds to flights being bought in October It is realistic that the minimum average cost is likely to occur just after the peak holiday summer season has finished.	1
	Total	**1**

5	$\mathrm{f}'(x) = -32x^{-5} + 1$	1
	Being parallel to *AB* means that gradient equals zero.	
	$-\dfrac{32}{x^5} + 1 = 0$	1
	$x^5 = 32 \Rightarrow x = 2$	1
	Total	**3**
6	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3}{2}x^{\frac{1}{2}} - 1$	1
	Gradient of normal $= -\dfrac{1}{5} \Rightarrow$ Gradient of tangent = 5	1
	$\dfrac{3}{2}x^{\frac{1}{2}} - 1 = 5$	
	$\dfrac{3}{2}x^{\frac{1}{2}} = 6 \Rightarrow x^{\frac{1}{2}} = 4 \Rightarrow x = 16$	1
	When $x = 16$, $y = -\dfrac{16}{5}$	1
	$y = -\dfrac{16}{5} \Rightarrow -\dfrac{16}{5} = 16\sqrt{16} - 16 + c \Rightarrow c = -\dfrac{256}{5}$	1
	Total	**5**
7	$\mathrm{f}'(x) = \dfrac{5}{3}x^{\frac{2}{3}} - \dfrac{4}{3}x^{\frac{1}{3}} + k^2$	1
	This is a quadratic in $x^{\frac{1}{3}}$ Discriminant of quadratic is $\dfrac{16}{9}k^2 - 4 \times \left(\dfrac{5}{3}\right) \times \left(k^2\right)$	1
	Discriminant equals $-\dfrac{44}{9}k^2$. This is negative (since $k \neq 0$, so quadratic has no roots).	1
	When $x = 0$, $\mathrm{f}'(x) = k^2$. This is positive and, because quadratic has no roots, we conclude that $\mathrm{f}'(x) > 0$ for all x. Therefore $\mathrm{f}(x)$ is increasing.	1
	Total	**4**
	TOTAL	**32**