

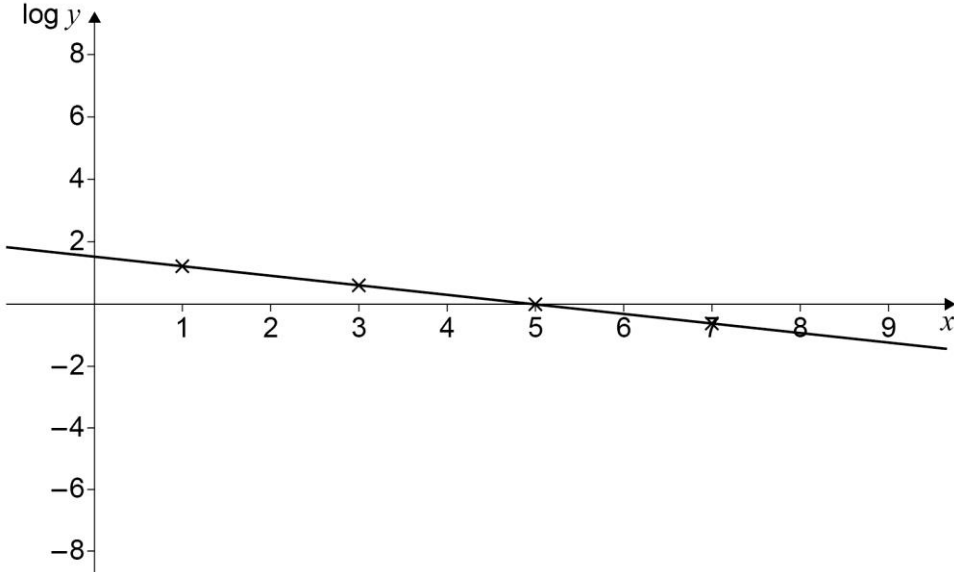
# AS and A-level MATHS

Ex ponentials II

Mark Sch eme

Specification content coverage: F5, F6, F7

Question	Solutions	Mark
1	$x = \frac{1}{2} \left( \frac{\log 6}{\log 4} - 5 \right)$	1
2	$(x + 2) \log 3 = \log 0.4$ $x = \frac{\log 0.4}{\log 3} - 2$ $x = -2.83$	1 1 1
3 (a)	$50 + 35e^0 = 85^\circ\text{C}$	1
3 (b)	Rate of change = $(35 \times -0.8)e^{-0.8t} = -28e^{-0.8t}$ when $t = 2$ rate of change = $-28e^{-0.8 \times 2} = -5.65$	1 1
3 (c)	$60 = 50 + 35e^{-0.8t}$ $\ln \left( \frac{10}{35} \right) = -0.8t$ $t = 1.565\dots \text{mins} = 94 \text{ seconds (nearest second)}$	1 1 1
3 (d)	For large values of $t$ , $T \approx 50$ this is too high for a room temperature	1 1

<p><b>4 (a)</b></p>	<p>Plot (for example) <math>\log y</math> against <math>x</math></p> <table border="1" data-bbox="312 387 1051 497"> <tr> <td><math>x</math></td> <td>1</td> <td>3</td> <td>5</td> <td>7</td> </tr> <tr> <td><math>\log y</math></td> <td>1.25</td> <td>0.646</td> <td>0.0453</td> <td>-0.558</td> </tr> </table>  <p><math>k</math> in range 30 – 36  <math>b</math> in range 0.45 – 0.55</p>	$x$	1	3	5	7	$\log y$	1.25	0.646	0.0453	-0.558	<p>1 1 1</p>
$x$	1	3	5	7								
$\log y$	1.25	0.646	0.0453	-0.558								
<p><b>4 (b)</b></p>	<p><math>y</math> in range 1.9 – 2.3</p>	<p>1</p>										
<p><b>5</b></p>	<p><math>240 = 200e^{2k}</math>  <math>2k = \ln \left( \frac{240}{200} \right)</math>  <math>k = \ln \frac{\left( \frac{6}{5} \right)}{2} \quad (0.0912)</math>  <math>y = 200e^{12\ln \frac{\left( \frac{6}{5} \right)}{2}}</math> or <math>240e^{10\ln \frac{\left( \frac{6}{5} \right)}{2}}</math>  <math>y = \text{£}597.20</math></p>	<p>1 1 1 1 1</p>										

<b>6</b>	$1.2 = 1.75e^{5k}$ $5k = \ln\left(\frac{1.2}{1.75}\right)$ $k = \ln\left(\frac{\frac{24}{35}}{5}\right) \quad (-0.0755)$ $0.8 = 1.75e^{(t+5)\ln\left(\frac{24}{35}\right)} \quad \text{or} \quad 0.8 = 1.2e^{t\ln\left(\frac{24}{35}\right)}$ $t = 10.37\dots \text{ hours or } t = 5.37\dots \text{ hours found}$ $t = 5 \text{ hours } 22 \text{ mins or } 322 \text{ mins (nearest minute)}$	1 1 1 1 1 1
<b>7</b>	$\text{gradient} = (10 - 2)/(3 - 1) (= 4)$ $\log y - 2 = 4(\log x - 1)$ $\log y = 4\log x - \log 100$ $y = \frac{x^4}{100}$ $y = \frac{10^4}{100} = 100$	1 1 1 1 1