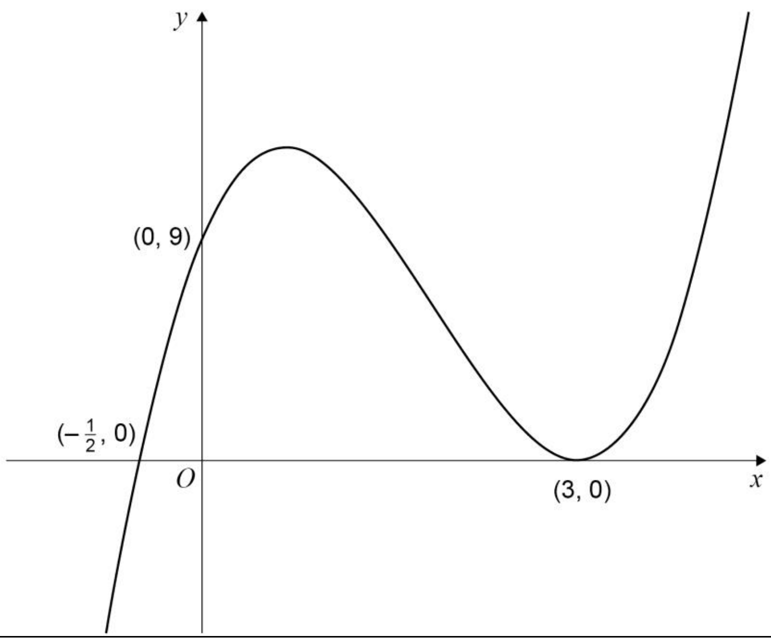
TOPIC TEST
MARK SCHEME

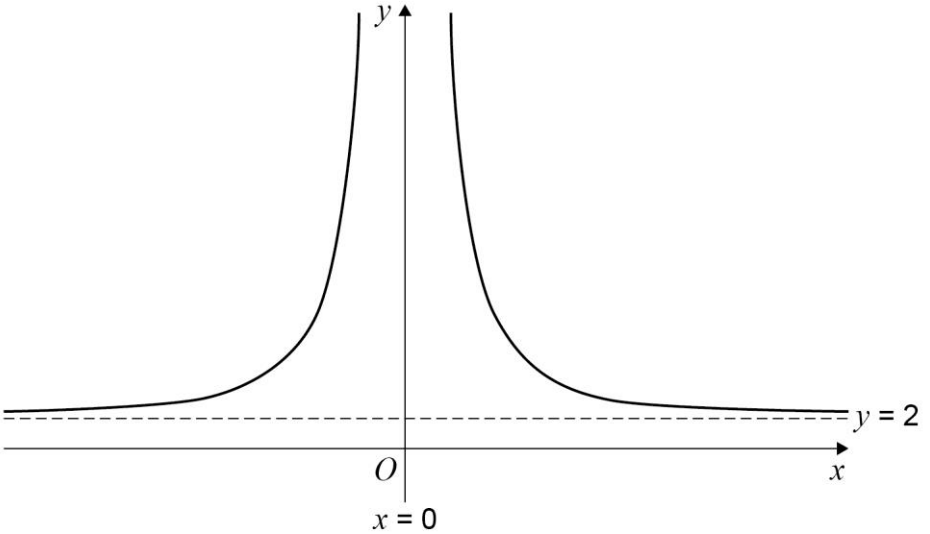
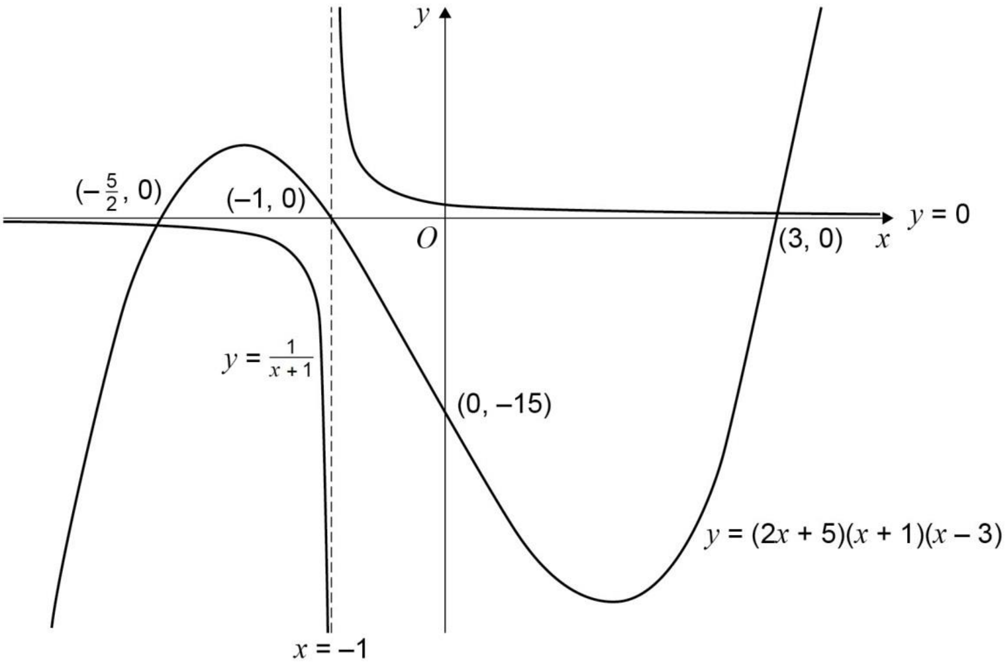
AS and A-level MATHS

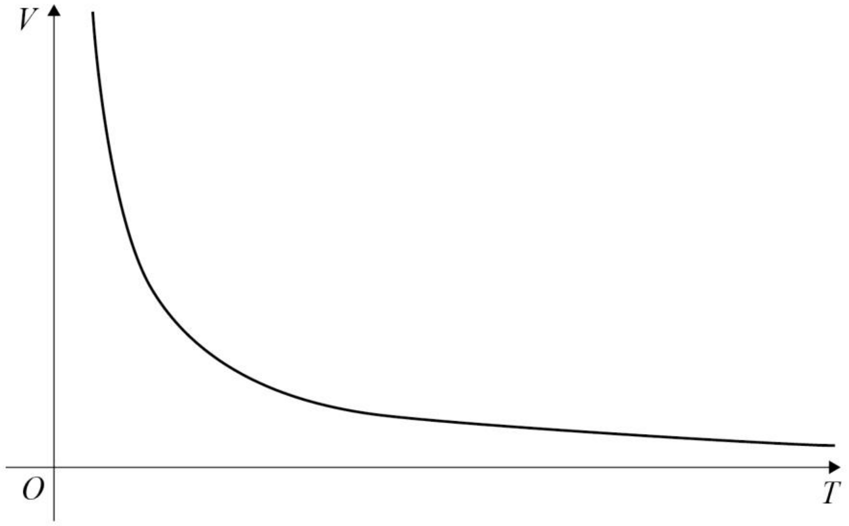
Graphs and transformations

Mark scheme

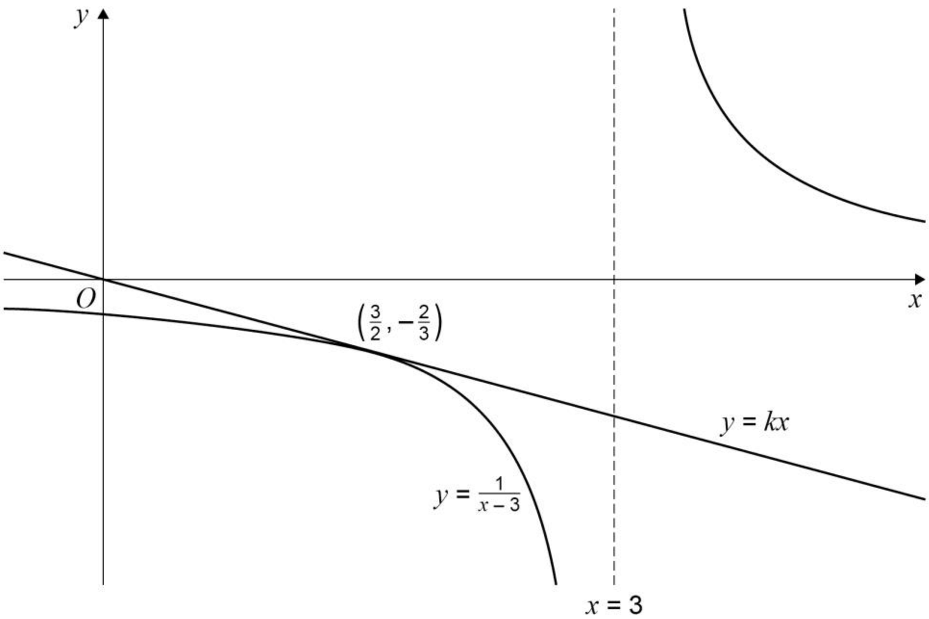
Specification content coverage: B7, B9

Question	Solutions	Mark
1	(c) A stretch, parallel to the y axis, scale factor $\frac{1}{4}$	1
2		1 Correct shape, including just touching x-axis 1 Correct intercepts

3 (a)	$y = \frac{5}{x^2} + 2$	1
3 (b)		1 correct shape 1 both asymptotes labelled correctly
4 (a)		1 shape of cubic curve 1 correct intercepts 1 shape of $y = \frac{1}{x+1}$ 1 asymptotes at $x = 1$ labelled correctly and curve approaching both asymptotes correctly
4 (b)	2 solutions	1

5 (a)	$V \propto \frac{1}{T} \quad \Rightarrow \quad V = \frac{k}{T}$ 	1 identifying relationship is a reciprocal functions 1 correct sketch, including approaching asymptotes
5 (b)	$6 = \frac{k}{70} \quad \Rightarrow \quad k = 420$ $5.6 = \frac{420}{T} \quad \Rightarrow \quad T = \frac{420}{5.6} = 75$ seconds	1 finding value of k (possibly implied) 1 finding T
6	New equation is $y = \frac{3}{\left(\frac{1}{4}x\right)^2}$ So $y = \frac{3}{\frac{1}{16}x^2} = 16 \times \frac{3}{x^2}$ Therefore $k = 16$	1 applying horizontal stretch correctly 1 Stating value of k
7	Replace x with $\left(\frac{x}{2}\right)$ Stretch, parallel to the x-axis, scale factor 2	1 "Stretch, parallel to x" 1 "Scale factor 2"
8 (a)	Replace x by $\frac{x}{3\sqrt{a}}$ Correctly stating $\left(\frac{x}{3\sqrt{a}}\right)^3 = \frac{x^3}{27a\sqrt{a}}$ and $\left(\frac{x}{3\sqrt{a}}\right)^2 = \frac{x^2}{9a}$ Correct derivation of $\mathrm{g}(x)$	1 1

	$54a\sqrt{a}\left(\frac{x}{3\sqrt{a}}\right)^3+27a\left(\frac{x}{3\sqrt{a}}\right)^2-51\sqrt{a}\left(\frac{x}{3\sqrt{a}}\right)+12$ $=54a\sqrt{a}\left(\frac{x^3}{27a\sqrt{a}}\right)+27a\left(\frac{x^2}{9a}\right)-51\sqrt{a}\left(\frac{x}{3\sqrt{a}}\right)+12$ $=2x^3+3x^2-17x+12$	1 No errors seen
8 (b)	Solutions to $\mathrm{g}(x)=0$ are x = –4, $x=\frac{3}{2}$ and x = 1. Roots of $\mathrm{f}(x)$ are found by dividing roots of $\mathrm{g}(x)$ by $3\sqrt{a}$ Solutions to $\mathrm{f}(x)=0$ are $x=\frac{-4}{3\sqrt{a}}$, $x=\frac{1}{2\sqrt{a}}$ and $x=\frac{1}{3\sqrt{a}}$	1 1 attempting to convert between roots of f and roots of g 1 all solutions corrects
9 (a)	Forming quadratic equal to zero $\frac{1}{x-3}=kx \quad \Rightarrow \quad 1=kx^2-3kx \quad \Rightarrow \quad 0=kx^2-3kx-1$ Putting discriminant equal to zero $9k^2-4(k)(-1)=0$ Solving $9k^2+4k=0 \Rightarrow \quad k=-\frac{4}{9} \quad (k\neq 0)$	1 1 1

9 (b)	Intersect when $0 = -\frac{4}{9}x^2 + \frac{4}{3}x - 1 \Rightarrow \quad x = \frac{3}{2}$ Point $\left(\frac{3}{2}, -\frac{2}{3}\right)$ marked on the graph 	1 1 correct shape of $\frac{1}{x-3} = 0$ 1 correct sketch of $y = -\frac{4}{9}x$, passing through origin and tangent to curve
9 (c)	The line $y = 0$ is an asymptote to $y = \frac{1}{x-3}$, hence this line won't intersect curve OR the equation $\frac{1}{x-3} = 0$ has no solutions.	1

Rationale

It is assumed that students are proficient at using calculator to solve cubic and quadratic equations.

15 marks scaffolded, with basic skills assessed
17 marks applying, including some more advanced problem-solving