TOPIC TEST
MARK SCHEME

AS and A-level MATHS

Integration

Mark scheme

Specification content coverage: H1, H2, H3

Question	Solutions	Mark
1 (a)	$y=\frac{3x^5}{5}-\frac{5x^2}{2}+2x+c$	1 One term correct 1 All correct, including $+c$
	Total	**2**
1 (b)	$12=\frac{3\left(2^5\right)}{5}-\frac{5\left(2^2\right)}{2}+2(2)+c$ $c=-\frac{6}{5}$ $y=\frac{3x^5}{5}-\frac{5x^2}{2}+2x-\frac{6}{5}$	1 Correctly substituting into equation 1
	Total	**2**
2 (a)	$2x^{-4}-7x^{-3}$	1
	Total	**1**
2 (b)	$\frac{2x^{-3}}{-3}+\frac{7x^{-2}}{2}+c$	1 One term correct 1 All correct, including $+c$ and two negatives becoming positive
	Total	**2**
3	$\int_1^4 6x^2-x^{\frac{1}{2}}\,\mathrm{d}x=\left[2x^3-\frac{2}{3}x^{\frac{3}{2}}\right]_1^4$ $=\left(2\left(4^3\right)-\frac{2}{3}\left(4^{\frac{3}{2}}\right)\right)-\left(2\left(1^3\right)-\frac{2}{3}\left(1^{\frac{3}{2}}\right)\right)$ $=\frac{364}{3}$	2 one mark each term 1 F(4) – F(1) 1 Correct answer
	Total	**4**

4	$\int_{-1}^{2} x^2 - 7\,dx = \left[\frac{x^3}{3} - 7x\right]_{-1}^{2}$	1 Correct integral
	$= \left(\frac{2^3}{3} - 7(2)\right) - \left(\frac{(-1)^3}{3} - 7(-1)\right)$	1 F(2) – F(–1)
	$= -18$	1: Correct answer
	Area $= 18$	1 Making negative value positive
	Total	**4**
5	Area under curve $= \int_{0}^{3} 2x^3 - 4x^2 + 5\,dx$	
	$= \left[\frac{2x^4}{4} - \frac{4x^3}{3} + 5x\right]_{0}^{3}$	1
	$= \left(\frac{2(3^4)}{4} - \frac{4(3)^3}{3} + 5(3)\right) - (0)$	
	$= \frac{39}{2}$	1
	Coordinates of A are (0,5). Coordinates of B are (3,23).	1
	Area of parallelogram $= \frac{1}{2} \times 3 \times (5 + 23) = 42$	1
	Shaded region= Parallelogram – area under curve $= 42 - \frac{39}{2} = \frac{45}{2}$	1
	Total	**5**
6 (a)	When $x = \frac{1}{4}$, gradient of tangent is $\frac{5}{2}$	
	$\frac{2}{\sqrt{\frac{1}{4}}} - p\sqrt{\frac{1}{4}} = \frac{5}{2}$	1
	$4 - \frac{1}{2}p = \frac{5}{2} \quad \Rightarrow \quad p = 3$	1
	Total	**2**

6 (b)	$f'(x) = 2x^{-\frac{1}{2}} - 3x^{\frac{1}{2}}$ $y = \dfrac{2x^{\frac{1}{2}}}{\frac{1}{2}} - \dfrac{3x^{\frac{3}{2}}}{\frac{3}{2}} + c = 4x^{\frac{1}{2}} - 2x^{\frac{3}{2}} + c$ When $x = \frac{1}{4}$, $y = \frac{5}{2}\left(\frac{1}{4}\right) + \frac{7}{8} = \frac{3}{2}$ So $\frac{3}{2} = 4\left(\frac{1}{4}\right)^{\frac{1}{2}} - 2\left(\frac{1}{4}\right)^{\frac{3}{2}} + c \quad \Rightarrow \quad c = -\frac{1}{4}$ Equation of curve is $y = 4x^{\frac{1}{2}} - 2x^{\frac{3}{2}} - \frac{1}{4}$	2 correct integral 1 Use equation of tangent to find the coordinates of a point on the curve 1
	Total	**4**
7	$f(x) = 8x^{-3} + 3x^{-2}$ $\int_1^k 8x^{-3} + 3x^{-2}\,dx = 6$ $\left[\dfrac{8x^{-2}}{-2} + \dfrac{3x^{-1}}{-1}\right]_1^k = 6$ $\left(-4k^{-2} - 3k^{-1}\right) - \left(-4 - 3\right) = 6$ $-\dfrac{4}{k^2} - \dfrac{3}{k} + 7 = 6$ $-\dfrac{4}{k^2} - \dfrac{3}{k} + 1 = 0$ Multiply by k^2 $k^2 - 3k - 4 = 0$ Solutions are $k = 4$ or $k = -1$ But k is a positive constant, so $k = 4$	1: Writing terms in xn form 2: Correct integral 1: Method F(k) – F(1) = 6 1: Method: Forming quadratic in k 1: Finding k
	Total	**6**
	TOTAL	**32**

Rationale

It is assumed that students are proficient at using calculator to solve quadratic equations and inequalities.
15 marks scaffolded, with basic skills assessed.
17 marks applying, including some more advanced problem-solving, basic proof and modelling questions.