Section 1: Motion in two dimensions

Solutions to Exercise level 1

1. (i) $\underset{\sim}{v} = \frac{d}{dt}\begin{pmatrix} 3t+1 \\ 2-t \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

(ii) $\underset{\sim}{a} = \frac{d}{dt}\begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \underset{\sim}{0}$

2. (i) $\underset{\sim}{v} = \int\left((3t+2)\underset{\sim}{i} + (2t^2 - 5)\underset{\sim}{j}\right)dt$

$= (\tfrac{3}{2}t^2 + 2t)\underset{\sim}{i} + (\tfrac{2}{3}t^3 - 5t)\underset{\sim}{j} + \underset{\sim}{c}$

When $t = 0, \underset{\sim}{v} = 0 \Rightarrow \underset{\sim}{c} = 0$

$\underset{\sim}{v} = (\tfrac{3}{2}t^2 + 2t)\underset{\sim}{i} + (\tfrac{2}{3}t^3 - 5t)\underset{\sim}{j}$

(ii) $\underset{\sim}{s} = \int\left((\tfrac{3}{2}t^2 + 2t)\underset{\sim}{i} + (\tfrac{2}{3}t^3 - 5t)\underset{\sim}{j}\right)dt$

$= (\tfrac{1}{2}t^3 + t^2)\underset{\sim}{i} + (\tfrac{1}{6}t^4 - \tfrac{5}{2}t^2)\underset{\sim}{j} + \underset{\sim}{d}$

When $t = 0, \underset{\sim}{s} = 0 \Rightarrow \underset{\sim}{d} = 0$

$\underset{\sim}{s} = (\tfrac{1}{2}t^3 + t^2)\underset{\sim}{i} + (\tfrac{1}{6}t^4 - \tfrac{5}{2}t^2)\underset{\sim}{j}$

3. (i) $\underset{\sim}{s} = \int\left(2t\underset{\sim}{i} + (3t^2 - 4t)\underset{\sim}{j}\right)dt$

$= t^2\underset{\sim}{i} + (t^3 - 2t^2)\underset{\sim}{j} + \underset{\sim}{c}$

When $t = 2, \underset{\sim}{s} = 7\underset{\sim}{i} + 4\underset{\sim}{j} \Rightarrow 7\underset{\sim}{i} + 4\underset{\sim}{j} = 4\underset{\sim}{i} + 0\underset{\sim}{j} + \underset{\sim}{c}$

$\Rightarrow \underset{\sim}{c} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$

$\underset{\sim}{s} = t^2\underset{\sim}{i} + (t^3 - 2t^2)\underset{\sim}{j} + 3\underset{\sim}{i} + 4\underset{\sim}{j}$

$= (t^2 + 3)\underset{\sim}{i} + (t^3 - 2t^2 + 4)\underset{\sim}{j}$

(ii) When $t = 5$, $\underset{\sim}{s} = (5^2 + 3)\underset{\sim}{i} + (5^3 - 2\times5^2 + 4)\underset{\sim}{j}$

$= 28\underset{\sim}{i} + 79\underset{\sim}{j}$

4. (i) $\underset{\sim}{u} = 15\underset{\sim}{i} + 20\underset{\sim}{j}$

$\underset{\sim}{a} = -10\underset{\sim}{j}$

Using the constant acceleration equations:

Velocity vector $\underset{\sim}{v} = \underset{\sim}{u} + \underset{\sim}{a}t = 15\underset{\sim}{i} + 20\underset{\sim}{j} - 10\underset{\sim}{j} \times t$

$= 15\underset{\sim}{i} + (20 - 10t)\underset{\sim}{j}$

Position vector $\underset{\sim}{r} = \underset{\sim}{u}t + \frac{1}{2}\underset{\sim}{a}t^2 = (15\underset{\sim}{i} + 20\underset{\sim}{j})t + \frac{1}{2} \times -10\underset{\sim}{j} \times t^2$

$= 15t\underset{\sim}{i} + (20t - 5t^2)\underset{\sim}{j}$

(ii) When the ball is moving in a direction of 45°, the components of the velocity in the $\underset{\sim}{i}$ and $\underset{\sim}{j}$ directions are equal.

$15 = 20 - 10t$

$10t = 5$

$t = 0.5$

(iii) When the ball returns to the ground, the $\underset{\sim}{j}$ component of the position vector is zero.

$20t - 5t^2 = 0$

$t(20 - 5t) = 0$

$t = 0$ or 4

It returns to the ground after 4 seconds.

5. (i) $\underset{\sim}{u} = 12\underset{\sim}{i}$

$\underset{\sim}{a} = -10\underset{\sim}{j}$

Using the constant acceleration equations:

Velocity $\underset{\sim}{v} = \underset{\sim}{u} + \underset{\sim}{a}t = 12\underset{\sim}{i} - 10\underset{\sim}{j} \times t$

$= 12\underset{\sim}{i} - 10t\underset{\sim}{j}$

Displacement $\underset{\sim}{s} = \underset{\sim}{u}t + \frac{1}{2}\underset{\sim}{a}t^2 = 12t\underset{\sim}{i} + \frac{1}{2} \times -10\underset{\sim}{j} \times t^2$

$= 12t\underset{\sim}{i} - 5t^2\underset{\sim}{j}$

(ii) When the stone lands, the $\underset{\sim}{j}$ component of the displacement is -10.

$-10 = -5t^2$

$t^2 = 2$

$t = \sqrt{2}$

The $\underset{\sim}{i}$ component of the displacement at this time is $12\sqrt{2}$.

The stone lands $12\sqrt{2}$ m from the base of the tower.

6. $\underset{\sim}{r} = 8t^3\underset{\sim}{i} + t^4\underset{\sim}{j}$

$\underset{\sim}{v} = \frac{d\underset{\sim}{r}}{dx} = 24t^2\underset{\sim}{i} + 4t^3\underset{\sim}{j}$

When $t = 2$, $\underset{\sim}{v} = 24 \times 2^2 \underset{\sim}{i} + 4 \times 2^3 \underset{\sim}{j}$
$= 96\underset{\sim}{i} + 32\underset{\sim}{j}$

$\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dx} = 48t\underset{\sim}{i} + 12t^2 \underset{\sim}{j}$
When $t = 2$, $\underset{\sim}{a} = 48 \times 2\underset{\sim}{i} + 12 \times 2^2 \underset{\sim}{j}$
$= 96\underset{\sim}{i} + 48\underset{\sim}{j}$

7. $\underset{\sim}{v} = 2t\underset{\sim}{i} + 3t^2 \underset{\sim}{j}$
$\underset{\sim}{s} = \int \underset{\sim}{v}\, dt = \int \left(2t\underset{\sim}{i} + 3t^2 \underset{\sim}{j}\right) dt$
$= t^2 \underset{\sim}{i} + t^3 \underset{\sim}{j} + \underset{\sim}{c}$
When $t = 0$, $\underset{\sim}{s} = \underset{\sim}{i} + 4\underset{\sim}{j} \Rightarrow \underset{\sim}{c} = \underset{\sim}{i} + 4\underset{\sim}{j}$
$\underset{\sim}{s} = t^2 \underset{\sim}{i} + t^3 \underset{\sim}{j} + \underset{\sim}{i} + 4\underset{\sim}{j}$
$= (t^2 + 1)\underset{\sim}{i} + (t^3 + 4)\underset{\sim}{j}$