

Section 1: Motion in two dimensions

Solutions to Exercise level 2

$$1. \quad \underline{r} = 2t^3 \underline{i} + 3t^2 \underline{j}$$

$$\underline{v} = \frac{d\underline{r}}{dt} = 6t^2 \underline{i} + 6t \underline{j}$$

$$\underline{a} = \frac{d\underline{v}}{dt} = 12t \underline{i} + 6 \underline{j}$$

$$\text{When } t = 3, \quad \underline{v} = 6 \times 3^2 \underline{i} + 6 \times 3 \underline{j} = 54 \underline{i} + 18 \underline{j}$$

$$\text{Magnitude of velocity} = \sqrt{54^2 + 18^2} = 56.9 \text{ ms}^{-1}.$$

$$\text{When } t = 3, \quad \underline{a} = 12 \times 3 \underline{i} + 6 \underline{j} = 36 \underline{i} + 6 \underline{j}$$

$$\text{Magnitude of acceleration} = \sqrt{36^2 + 6^2} = 36.5 \text{ ms}^{-2}.$$

$$2. \quad \underline{F} = m\underline{a}$$

$$4t \underline{i} + 6 \underline{j} = 2\underline{a}$$

$$\underline{a} = 2t \underline{i} + 3 \underline{j}$$

$$\underline{v} = \int \underline{a} dt = \int (2t \underline{i} + 3 \underline{j}) dt = t^2 \underline{i} + 3t \underline{j} + \underline{c}$$

$$\text{When } t = 0, \quad \underline{v} = 5 \underline{j} \text{ so } \underline{c} = 5 \underline{j}$$

$$\underline{v} = t^2 \underline{i} + 3t \underline{j} + 5 \underline{j} = t^2 \underline{i} + (3t + 5) \underline{j}$$

$$\text{When } t = 3, \quad \underline{v} = 3^2 \underline{i} + (3 \times 3 + 5) \underline{j}$$

$$= 9 \underline{i} + 14 \underline{j}$$

$$\underline{r} = \int \underline{v} dt = \int (t^2 \underline{i} + (3t + 5) \underline{j}) dt = \frac{1}{3} t^3 \underline{i} + \left(\frac{3}{2} t^2 + 5t \right) \underline{j} + \underline{d}$$

$$\text{When } t = 0, \quad \underline{r} = 0 \text{ so } \underline{d} = 0$$

$$\underline{r} = \frac{1}{3} t^3 \underline{i} + \left(\frac{3}{2} t^2 + 5t \right) \underline{j}$$

$$\text{When } t = 3, \quad \underline{r} = \frac{1}{3} \times 3^3 \underline{i} + \left(\frac{3}{2} \times 3^2 + 5 \times 3 \right) \underline{j}$$

$$= 9 \underline{i} + \frac{57}{2} \underline{j}$$

$$3. \quad \underline{r} = 6t \underline{i} - 4t^2 \underline{j}$$

$$\underline{v} = \frac{d\underline{r}}{dt} = 6 \underline{i} - 8t \underline{j}$$

$$\underline{a} = \frac{d\underline{v}}{dt} = -8 \underline{j}$$

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$$\underline{F} = m\underline{a} = 4 \times -8\underline{j} = -32\underline{j}$$

The magnitude of F is 32.

4. (i) $\underline{r} = (2t - 1)\underline{i} - t^2\underline{j}$

$$\underline{v} = \frac{d\underline{r}}{dt} = 2\underline{i} - 2t\underline{j}$$

(ii) When $t = 0$, $\underline{v} = 2\underline{i}$

so the initial direction of motion is in the positive \underline{i} direction.

(iii) $\underline{a} = \frac{d\underline{v}}{dt} = -2\underline{j}$ which is not dependent on time, so the acceleration is constant.

(iv) The acceleration acts only in the \underline{j} direction, so the component of the velocity in the \underline{i} direction is constant. Since the initial velocity is $2\underline{i}$, the component of the velocity in the \underline{i} direction can never be zero, and so the motion can never be in the \underline{j} direction.

(v) $x = 2t - 1 \Rightarrow t = \frac{1}{2}(x + 1)$

$$y = -t^2 = -\frac{1}{4}(x + 1)^2$$

The equation of the path is $4y + (x + 1)^2 = 0$

5. (i) Resultant force $= \underline{F} + \underline{T} = 12\underline{i} - 2\underline{j} + 10\underline{i} + 12\underline{j}$
 $= 22\underline{i} + 10\underline{j}$

(ii) $\underline{F} = m\underline{a}$
 $22\underline{i} + 10\underline{j} = 20\underline{a}$

$$\underline{a} = 1.1\underline{i} + 0.5\underline{j}$$

(iii) $\underline{u} = 0$, $\underline{a} = 1.1\underline{i} + 0.5\underline{j}$

Using constant acceleration equation:

$$\text{Displacement } \underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2$$

$$= 0 + \frac{1}{2}(1.1\underline{i} + 0.5\underline{j})t^2 = 0.55t^2\underline{i} + 0.25t^2\underline{j}$$

Initial position is $3\underline{j}$,

$$\text{so position vector is } \underline{r} = 3\underline{j} + 0.55t^2\underline{i} + 0.25t^2\underline{j}$$

$$= 0.55t^2\underline{i} + (3 + 0.25t^2)\underline{j}$$

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$$6. \quad \underline{v} = \frac{d\underline{r}}{dt} = (2t - 4)\underline{i} + (3t^2 + 2ft)\underline{j}$$

Since the particle comes to instantaneous rest, there is a value for t for which

$$2t - 4 = 0 \text{ and } 3t^2 + 2ft = 0$$

$$2t - 4 = 0 \Rightarrow t = 2$$

$$\text{When } t = 2, 3t^2 + 2ft = 0 \Rightarrow 12 + 4f = 0 \Rightarrow f = -3$$

$$7. \quad \underline{a} = \frac{d\underline{v}}{dt} = 2t\underline{i} + 3\underline{j}$$

$$\text{When } t = 3, \underline{a} = 6\underline{i} + 3\underline{j}$$

$$\begin{aligned} \underline{s} &= \int \underline{v} dt = \int (t^2\underline{i} + 3t\underline{j}) dt \\ &= \frac{1}{3}t^3\underline{i} + \frac{3}{2}t^2\underline{j} + \underline{c} \end{aligned}$$

$$\text{When } t = 0, \underline{s} = 18\underline{i} - 24\underline{j} \Rightarrow \underline{c} = 18\underline{i} - 24\underline{j}$$

$$\begin{aligned} \underline{s} &= \frac{1}{3}t^3\underline{i} + \frac{3}{2}t^2\underline{j} + 18\underline{i} - 24\underline{j} \\ &= \left(\frac{1}{3}t^3 + 18\right)\underline{i} + \left(\frac{3}{2}t^2 - 24\right)\underline{j} \end{aligned}$$

$$\begin{aligned} \text{When } t = 3, \underline{s} &= (9 + 18)\underline{i} + \left(\frac{27}{2} - 24\right)\underline{j} \\ &= 27\underline{i} - 10.5\underline{j} \end{aligned}$$