Section 1: Motion in two dimensions

Solutions to Exercise level 3

1. (i) If the particle were at)

$1-t^2=0 \quad \Rightarrow (t-1)(t+1)=0 \quad \Rightarrow t=\pm 1$

and $t^2-3t=0 \quad \Rightarrow t(t-3)=0 \quad \Rightarrow t=0$ or 3

These are incompatible so the particle is never at the origin.

(ii) $\underline{v}=\frac{d\underline{r}}{dt}=\begin{pmatrix}-2t \\ 2t-3\end{pmatrix}$

When the particle is moving parallel to the y-axis, the x-component of the velocity is zero (and the y-component is not zero)

$\Rightarrow -2t=0$

$\Rightarrow t=0$

When $t=0$, $2t-3=-3$

so the particle is moving parallel to the y-axis when $t=0$.

(iii) If the velocity is perpendicular to the position vector,

$$\frac{2t-3}{-2t}\times\frac{t^2-3t}{1-t^2}=-1$$

$$(2t-3)(t^2-3t)=2t(1-t^2)$$

$$2t^3-9t^2+9t=2t-2t^3$$

$$4t^3-9t^2+7t=0$$

$$t(4t^2-9t+7)=0$$

If $t\neq 0$, then $4t^2-9t+7=0$

Discriminant $=81-4\times 4\times 7=-31$, so quadratic equation has no roots

so velocity is never perpendicular to its position vector for $t\neq 0$.

(iv) If velocity is parallel to position vector,

$$\frac{2t-3}{-2t}=\frac{t^2-3t}{1-t^2} \quad (t\neq 0,1,-1)$$

$$(2t-3)(1-t^2)=-2t(t^2-3t)$$

$$-2t^3+3t^2+2t-3=-2t^3+6t^2$$

$$3t^2-2t+3=0$$

Discriminant $=4-4\times 3\times 3=-32$ so equation has no roots

(Note: when $t=0,1$ or -1 the gradients are never undefined together).

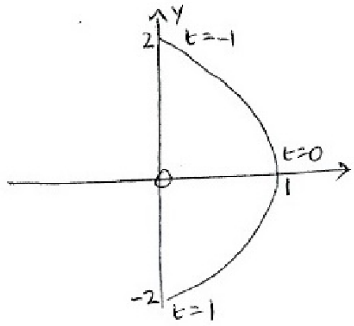
(v) Distance from origin is given by

$$d^2 = (1-t^2)^2 + (t^2-3t)^2$$
$$= 1 - 2t^2 + t^4 + t^4 - 6t^3 + 9t^2$$
$$= 2t^4 - 6t^3 + 7t^2 + 1$$
$$\frac{d}{dx}\left(d^2\right) = 8t^3 - 18t^2 + 14t$$
$$= 2t(4t^2 - 9t + 7)$$

At minimum distance, $\frac{d}{dx}\left(d^2\right) = 0$

Since the quadratic has no roots, $\frac{d}{dx}\left(d^2\right) = 0$ when $t = 0$ only

When $t < 0$, $\frac{d}{dx}\left(d^2\right) < 0$, and when $t > 0$, $\frac{d}{dx}\left(d^2\right) > 0$,
so d^2 is increasing, and therefore $t = 0$ is a minimum.
So the minimum distance is 1.



2. (i) $x = 5\cos t$

$y = 5\sin t$

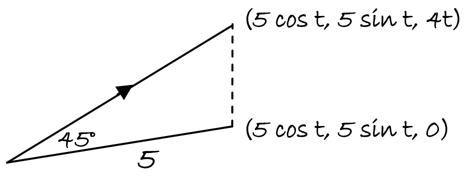
$\Rightarrow x^2 + y^2 = 25(\cos^2 t + \sin^2 t) = 25$

So from above the motion is seen as a circle, with the particle moving anticlockwise. However z increases as time increases, so the motion is a helix (it spirals upwards).

(ii) Distance from O $= \sqrt{x^2 + y^2 + z^2}$

$= \sqrt{25 + 16t^2}$

(iii)



$\tan 45° = 1 \quad \Rightarrow$ z-coordinate must be 5

so $4t = 5 \quad \Rightarrow t = 1.25$

AQA A level Maths Kinematics 1 Exercise solutions

3. Taking the position of the competitor as the origin:

For target: $\underline{r} = \begin{pmatrix} l \\ vt - \frac{1}{2}gt^2 \end{pmatrix}$

For bullet (fired time T later): $\underline{r} = \begin{pmatrix} u(t-T)\cos\alpha \\ u(t-T)\sin\alpha - \frac{1}{2}g(t-T)^2 \end{pmatrix}$

If the bullet and target collide:

$$\begin{pmatrix} l \\ vt - \frac{1}{2}gt^2 \end{pmatrix} = \begin{pmatrix} u(t-T)\cos\alpha \\ u(t-T)\sin\alpha - \frac{1}{2}g(t-T)^2 \end{pmatrix}$$

$$l = u(t-T)\cos\alpha \quad \Rightarrow t - T = \frac{l}{u\cos\alpha} \quad \Rightarrow t = T + \frac{l}{u\cos\alpha}$$

$$vt - \tfrac{1}{2}gt^2 = u(t-T)\sin\alpha - \tfrac{1}{2}g(t-T)^2$$

$$v\left(T + \frac{l}{u\cos\alpha}\right) - \tfrac{1}{2}g\left(T + \frac{l}{u\cos\alpha}\right)^2 = u\frac{l}{u\cos\alpha}\sin\alpha - \tfrac{1}{2}g\left(\frac{l}{u\cos\alpha}\right)^2$$

$$vT + \frac{vl}{u\cos\alpha} - \tfrac{1}{2}gT^2 - \frac{gTl}{u\cos\alpha} - \frac{gl^2}{2u^2\cos^2\alpha} = \frac{l\sin\alpha}{\cos\alpha} - \frac{gl^2}{2u^2\cos^2\alpha}$$

$$vT - \tfrac{1}{2}gT^2 + \frac{vl}{u\cos\alpha} - \frac{gTl}{u\cos\alpha} = \frac{l\sin\alpha}{\cos\alpha}$$

$$\left(vT - \tfrac{1}{2}gT^2\right)\cos\alpha + \frac{vl - gTl}{u} = l\sin\alpha$$

$$l\sin\alpha - \frac{v - gT}{u}l = \left(vT - \tfrac{1}{2}gT^2\right)\cos\alpha$$