

Section A: The basics

- 1 (a) Circle the equivalent expression to $(\sqrt[3]{x})^4$
- $x^{\frac{3}{4}}$ $\frac{4}{\sqrt[3]{x}}$ $x^{\frac{4}{3}}$ $x^{\frac{4}{3}}$
- [1 mark]**
- 1 (b) Write down the value of p and q given that:
- (i) $9^p = 1$
- (ii) $\frac{1}{81} = 3^q$
- [2 marks]**
- 2 (a) Simplify the expression $(5 + \sqrt{2})(5 - 2\sqrt{2})$
- [2 marks]**
- 2 (b) Simplify $\frac{4}{\sqrt{5}-3}$ by rationalising the denominator
- [3 marks]**
- 3 (a) Factorise the expression $x^2 - 2$
- [1 mark]**
- 3 (b) Factorise the expression $6x^2 - 5x - 21$
- [2 marks]**
- 4 (a) Sketch the curve $y = x^2 - x - 4$, indicating the exact intercepts on the x -axis and the y -axis.
- [4 marks]**
- 4 (b) Complete the square on the curve $y = x^2 - x - 4$
- [2 marks]**
- 4 (c) State the minimum point and any lines of symmetry
- [2 marks]**

Section B: A bit more thinking

- 5 A parallelogram has base $(3 + 5\sqrt{2})$ cm and area $(10 + 3\sqrt{2})$ cm².

Algebraic Manipulation, Quadratic Equations

Find the perpendicular height in its simplest form

[4 marks]

6 Solve the quadratic equation $\frac{5}{(x+3)} + \frac{1}{(x-1)} = 2$

[4 marks]

7 Solve $x^{\frac{2}{3}} - 17x^{\frac{1}{3}} + 16 = 0$

[5 marks]

Simultaneous equations, linear and quadratic inequalities

Section A: The basics

- 1** Calculate the discriminant of the quadratic $y = 3x^2 - 5x + 4$.
Use this to determine how many solutions the equation has.
[3 marks]
- 2** Solve the inequality $3x + 4 < 5x - 2$
[2 marks]
- 3** Find the points of intersection between the graph $y = x^2 + 5x - 3$ and the line $y = 2x + 1$.
[3 marks]
- 4** Expand $(2x + 3)(x^2 - 3x + 6)$
[2 marks]
- 5 (a)** Using the Factor Theorem, show that $x - 2$ is a factor of $p(x) = x^3 + 4x^2 - 19x + 14$
[2 marks]
- (b)** Hence, find the quotient and express $x^3 + 4x^2 - 19x + 14$ as a product of 3 linear factors.
[3 marks]

Section B: A bit more thinking

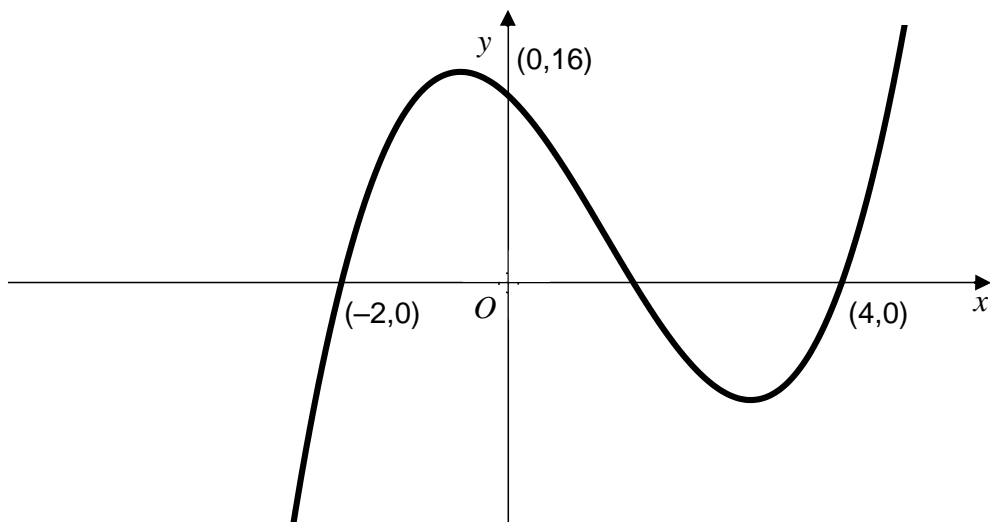
- 6 Given that $x + 2$ and $x - 3$ are factors of the polynomial $p(x) = 3x^3 + bx^2 - cx + 5$, find the value of b and c .

[4 marks]

- 7 Prove that the line $y = x - 5$ does not intersect with the quadratic $y = x^2 + 6x + 13$

[4 marks]

- 8 Use the sketch below to identify the cubic equation in the form $y = x^3 + ax^2 + bx + c$, where a , b and c are integers.



[4 marks]

- 9 Given that the cubic has a factor of $x - 2$, solve the inequality $x^3 + 2x^2 - 13x + 10 > 0$

[5 marks]

Section A: The basics

- 1 Which of the following options represents the transformation that maps the graph of $y = |x + 2|$ onto the graph of $y = \frac{1}{4}|x + 2|$?

Circle your answer.

A stretch, parallel to the x -axis, scale factor $\frac{1}{4}$

A stretch, parallel to the x -axis, scale factor 4

A stretch, parallel to the y -axis, scale factor $\frac{1}{4}$

A stretch, parallel to the y -axis, scale factor 4

[1 marks]

- 2 Sketch the graph of the curve with equation $y = (2x + 1)(x - 3)^2$

[2 marks]

- 3 (a) The curve with equation $y = \frac{5}{x^2}$ is translated by vector $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$.

Write down the equation of the transformed curve.

[1 mark]

- 3 (b) Sketch the transformed curve, labelling the equations of its asymptotes

[2 marks]

- 4 (a) Sketch, on the same set of axes, the graphs of the curves $y = \frac{1}{x+1}$ and $y = (2x + 5)(x + 1)(x - 3)$.

Label the points where the curves cross the axes, where the curves intersect and any asymptotes.

[4 marks]

- 4 (b) Hence state the number of real solutions to the equation

$$\frac{1}{x+1} = (2x + 5)(x + 1)(x - 3)$$

[1 mark]

Graphs and Transformations

- 5 (a)** The average velocity, V metres per second, at which a runner completes a race is inversely proportional to the total time, T seconds, it takes for them to complete the race.

Sketch a graph of V against T for positive values of T

[2 marks]

- 5 (b)** Alex completes the race in 70 seconds and his average velocity is 6 metres per second.

Find the time it takes Dexter to complete the race, given that his average speed is 5.6 metres per second.

[2 marks]

Section B: A bit more thinking

6 The curve with equation $y = \frac{3}{x^2}$ is stretched by scale factor 4 parallel to the x -axis.

This transformed curve could also be obtained by stretching $y = \frac{3}{x^2}$ by scale factor k parallel to the y -axis.

State the value of k .

[2 marks]

7 State the transformation that maps the graph of $y = 4x^2 + 3$ onto the graph of $y = x^2 + 3$

[2 marks]

8 (a) $f(x) = 54a\sqrt{a}x^3 + 27ax^2 - 51\sqrt{a}x + 12$, where a is a non-zero constant.

Show that a stretch scale factor $3\sqrt{a}$ parallel to the x -axis transforms the curve $y = f(x)$ onto the curve $y = g(x)$, where $g(x) = 2x^3 + 3x^2 - 17x + 12$

[3 marks]

8 (b) Hence find the solutions to $f(x) = 0$, giving your answers in terms of a .

[3 marks]

9 (a) The line $y = kx$, where k is a non-zero constant, touches the curve $y = \frac{1}{x-3}$ at exactly one point.

Find the value of k .

[3 marks]

9 (b) Using the value of k found in part **(a)**, sketch the curves $y = \frac{1}{x-3}$ and $y = kx$ on the same set of axes.

You should label the point of intersection between the two curves.

[3 marks]

9 (c) Explain why it is necessary to have the condition that k is a non-zero constant.

[1 mark]

Section A: The basics

- 1 Write down the equation of the line with gradient 3 through the point $(4, -1)$ in the form $y - y_1 = m(x - x_1)$. **[1 mark]**
- 2 (a) Find the gradient and the x - and y -intercepts of the line with equation $2x + 3y - 7 = 0$. **[3 marks]**
- 2 (b) Find the equation of a line that is parallel to $2x + 3y - 7 = 0$ through the point $(3, 5)$. **[3 marks]**
- 2 (c) When does the line $y = 2x + 1$ intersect the line $2x + 3y - 7 = 0$? **[2 marks]**
- 3 (a) The point P has coordinates $(-2, 2)$, the point Q has coordinates $(1, 6)$ and the point R has coordinates $(5, 3)$.
Show that PQ and QR are perpendicular. **[3 marks]**
- 3 (b) Find the distance of RQ . **[2 marks]**
- 4 Find the equation of a circle with centre $(2, -5)$ and diameter 16. **[2 marks]**
- 5 Find the centre and radius of the circle with equation $x^2 + y^2 + 4x - 6y - 8 = 0$. **[3 marks]**

Section B: A bit more thinking

- 6** A circle passes through points $A(7, 4)$, $B(10, 6)$ and $C(12, 3)$. Show that AC must be the diameter of the circle.
- [4 marks]**
- 7** The points $A(1, 4)$ and $B(5, 1)$ lie on a circle. The line segment AB is a chord. Find the equation of a diameter of the circle.
- [5 marks]**
- 8** A line has equation $y = k$, where k is a constant. For which values of k does the line not intersect the circle with equation $x^2 + 3x + y^2 + 2y - \frac{3}{4} = 0$.
- [4 marks]**

Section A: The basics

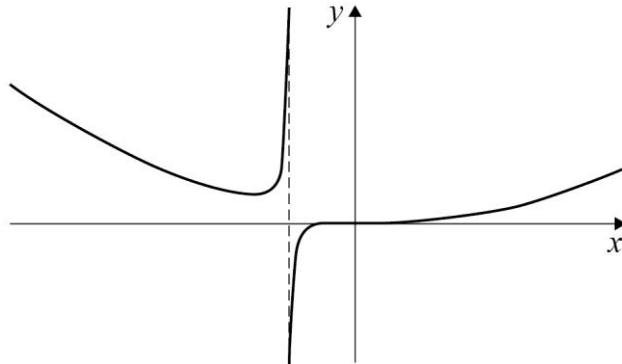
- 1 (a) Express the first four terms of the expansion $(2 - 3x)^8$ in ascending powers of x .
[4 marks]
- 2 Find the coefficient of x^5 in the expansion $\left(2 + \frac{1}{2}x\right)^{11}$.
[3 marks]
- 3 (a) By first completing the square, find the minimum point for the quadratic $y = x^2 + 7x - 30$.
[3 marks]
- 3 (b) Hence, sketch the graph $y = x^2 + 7x - 30$, including labelling where it crosses the axes.
[4 marks]

Section B: A bit more thinking

- 4 (a) Using the Factor Theorem, show that $x + 3$ is a factor of $p(x) = x^3 + 9x^2 + 27x + 27$
[2 marks]
- 4 (b) Hence, or otherwise express $x^3 + 9x^2 + 27x + 27$ as a product of linear factors.
[2 marks]
- 4 (c) Hence, describe the transformation from $y = x^3 + 9x^2 + 27x + 27$ onto $y = (x - 2)^3 + 2$
[3 marks]
- 5 (a) Using the binomial expansion, or otherwise, express $(2 + 3x)^3$ in the form $a + bx + cx^2 + x^3$.
[2 marks]
- 5 (b) Hence find the coefficient of x in the expansion $(2 + 3x)^3(1 - x^{-1})^5$.
[4 marks]
- 6 Show with clear steps and a clear conclusion whether the line $y = 3x + 2$ is a tangent to the circle $(y - 1)^2 + (x - 3)^2 = 25$.
[5 marks]

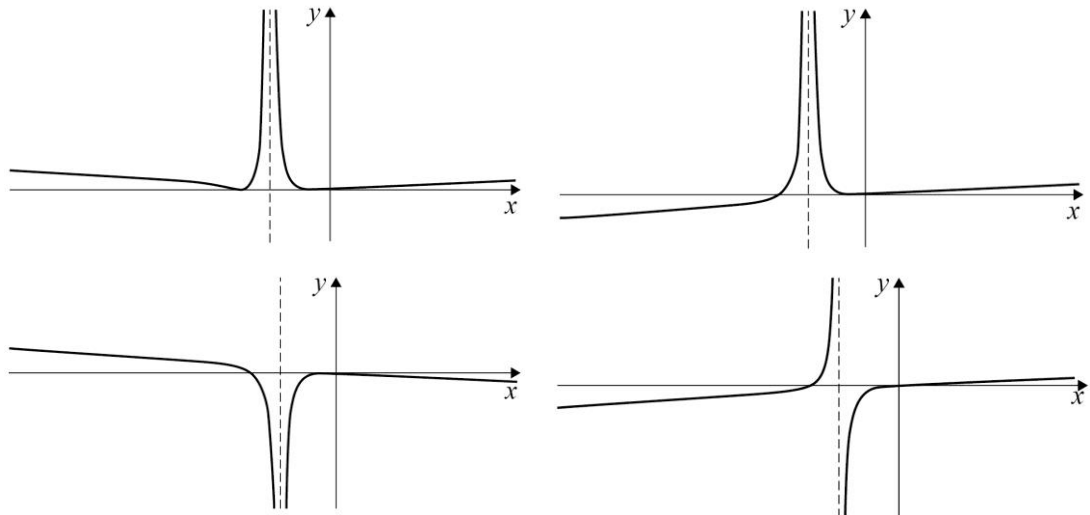
Section A: The basics

- 1 The graph of $y = f(x)$ is shown in the diagram.



Which of the following graphs is a sketch of its gradient function?

Circle your answer



[1 mark]

- 2 (a) Use the binomial expansion to express $(2+h)^3$ in the form $h^3 + 6h^2 + ph + q$, where p and q are integers to be determined.

[2 marks]

- 2 (b) The curve with equation $y = x^3 - 5x$ passes through the point $P(2, -2)$ and the point Q , where $x = 2 + h$.

Using differentiation from first principles, find the gradient of the tangent to the curve at the point P .

[3 marks]

Differentiation 1

3 (a) Find $\frac{dy}{dx}$ when $y = x^4 - 3x^3 + 2x + 14$

[2 marks]

3 (b) Find $\frac{d^2y}{dx^2}$

[1 mark]

4 Given $f(x) = (2x - 3)(4 + 7x - 2x^2)$, find $f'(x)$.

[3 marks]

5 (a) Express $x^2\sqrt{x}$ in the form x^p , where p is a rational number.

[1 mark]

5 (b) Hence find $\frac{dy}{dx}$ when $y = x^2\sqrt{x} - \frac{3}{x}$.

[2 marks]

Section B: A bit more thinking

- 6 Samir, a maths student, is using differentiation from first principles to evaluate the gradient of the curve $y = x^2 + 6x$ at the point where $x = -4$.

His working is as follows:

Step 1 When $x = -4$, $y = 16 - 24 = -8$

$$y = (-4 + h)^2 + 6(-4 + h)$$

Step 2 When $x = -4 + h$, $= 16 + h^2 - 24 + 6h$
 $= h^2 + 6h - 8$

Step 3 Gradient of chord $= \frac{(h^2 + 6h - 8) - (-8)}{(-4 + h) - (-4)} = \frac{h^2 + 6h}{h} = h + 6$

Step 4 Let $h = 0$. Then $\frac{dy}{dx} = 6$

There are two errors in Samir's working. Explain where these two errors occur.

[2 marks]

- 7 (a) The height of a mountain, h kilometres, at a horizontal distance of x kilometres from a starting point, can be modelled by the equation $h = 0.75x^{\frac{4}{3}} - 0.3x^2$.

Find $\frac{dh}{dx}$ when $x = 1$ and interpret this value in context.

[3 marks]

- 7 (b) Find $\frac{d^2h}{dx^2}$ when $x = 1$ and interpret this value in context.

[2 marks]

- 7 (c) State one limitation of using the equation $h = 0.75x^{\frac{4}{3}} - 0.3x^2$ to measure the actual height of the mountain when at a horizontal distance of x kilometres from the starting point.

[1 mark]

- 8 Given $f(x) = (5 + 2\sqrt{x})^3$, find $f'(x)$.

[4 marks]

- 9 Find the coordinates of the points on the curve $y = 3x^3 + \frac{4}{x}$ where the gradient is equal to -35 .

[5 marks]

Section A: The basics

- 1 (a) Find the gradient of the curve $y = \sqrt{x} + \frac{1}{18}x^3$ at the point where $x = 9$.
[3 marks]
- 1 (b) Hence find an equation for the normal to the curve at the point $\left(9, \frac{87}{2}\right)$.
[2 marks]
- 2 (a) Given $y = 12x^2 - \frac{3}{x} + \frac{4}{5}$, find an expression for $\frac{dy}{dx}$.
[2 marks]
- 2 (b) Hence find the x -coordinate of the stationary point.
[2 marks]
- 2 (c) Find an expression for $\frac{d^2y}{dx^2}$ and hence determine the nature of the stationary point.
[2 marks]
- 3 (a) Given $f(x) = 2x^3 + 2x^2 - 42x + 15$, show that the values of x for which $f(x)$ is decreasing satisfy the inequality $3x^2 + 2x - 21 < 0$.
[2 marks]
- 3 (b) Hence find the values of x for which $f(x)$ is decreasing.
[2 marks]

Section B: A bit more thinking

- 4 (a) The average cost, $\pounds P$, of purchasing a return flight from Manchester to Greece t months after 1st January can be modelled by the equation:

$$P = 0.67t^3 - 13.32t^2 + 70t + 207$$

Use calculus to find the value of t at which the minimum average cost occurs and justify that it is a minimum.

[4 marks]

- 4 (b) Explain whether the value of t found in part (a) is realistic.

[1 mark]

- 5 The function $f(x) = \frac{8}{x^4} + x - 7$ has positive roots at the points A and B .

Find the x -coordinate of the point on the curve where the gradient is parallel to AB .

[3 marks]

- 6 The line $y = -\frac{1}{5}x$ is a normal to the curve $y = x\sqrt{x} - x + c$. Find the value of c .

[5 marks]

- 7 Prove that the function $f(x) = x^{\frac{5}{3}} - kx^{\frac{4}{3}} + k^2x$ is increasing for all non-zero values of the real constant k .

[4 marks]

Integration

Section A: The basics

- 1 (a) A curve has gradient function given by $\frac{dy}{dx} = 3x^4 - 5x + 2$

Find the general equation for this curve, giving your answer in the form $y = f(x)$.

[2 marks]

- 1 (b) Given further that the curve passes through the point (2, 12), find the equation for the curve, giving your answer in the form $y = f(x)$.

[2 marks]

- 2 (a) Express $\frac{2x - 7x^2}{x^5}$ in the form $2x^p - 7x^q$

[1 mark]

- 2 (b) Hence find $\int \frac{2x - 7x^2}{x^5} dx$

[2 marks]

- 3 Evaluate $\int_1^4 6x^2 - \sqrt{x} dx$. You must show your working.

[4 marks]

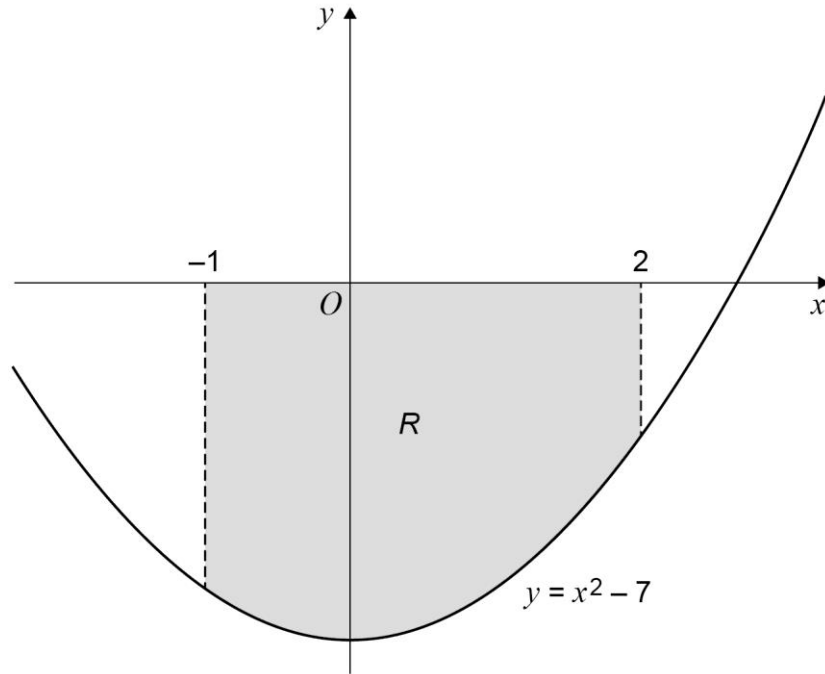
Integration

4

The diagram shows a shaded region R bounded by the curve $y = x^2 - 7$, the x -axis and the lines $x = -1$ and $x = 2$.

Find the area of R .

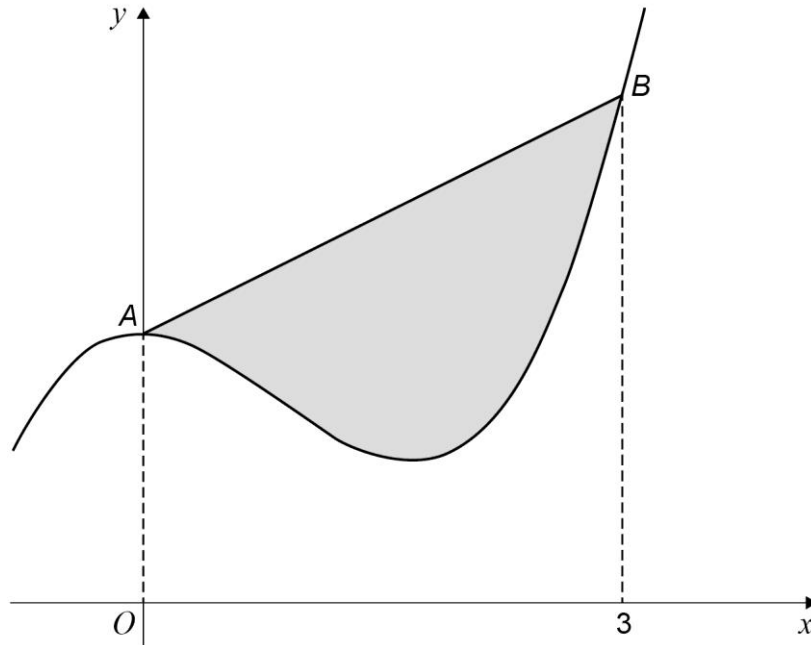
Fully justify your answer.



[4 marks]

Section B: A bit more thinking

- 5 The diagram shows the graph of the curve
 $y = 2x^3 - 4x^2 + 5$
 The curve intersects the y -axis at the point A .
 The point B lies on the curve when $x = 3$



Find the area of the shaded region bounded by the curve and the line segment AB .
 Fully justify your answer.

[5 marks]

- 6 (a) The curve $y = f(x)$ has a gradient function

$$f'(x) = \frac{2}{\sqrt{x}} - p\sqrt{x}$$

A is the point on the curve where $x = \frac{1}{4}$

The equation of the tangent to the curve at A is $y = \frac{5}{2}x + \frac{7}{8}$

Show that $p = 3$.

Fully justify your answer

[2 marks]

- 6 (b) Find the equation of the curve, giving your answer in the form $y = f(x)$

[4 marks]

Integration

7

$$f(x) = \frac{8+3x}{x^3}$$

Find the value of the positive constant k such that $\int_1^k f(x) dx = 6$.

Fully justify your answer.

[6 marks]

Trigonometry 1

Section A: The basics

- 1 (a) Circle the function that has a vertical line of symmetry at $y = 450^\circ$.
- $y = \sin x$ $y = \cos x$ $y = \tan x$
- [1 mark]**
- 1 (b) Describe the symmetry and periodicity of the graph $y = \tan x$.
- [2 marks]**
- 2 Given that $\cos \theta = \frac{3}{5}$ and that θ is acute, find the exact values of $\sin \theta$ and $\tan \theta$.
- [2 marks]**
- 3 (a) Sketch the graph of $y = \cos x$ for $-180^\circ \leq x \leq 180^\circ$.
- [2 marks]**
- 3 (b) Solve the equation $\cos x = -\frac{3}{4}$, giving all the answers between $0^\circ \leq x \leq 360^\circ$ correct to three significant figures.
- [2 marks]**
- 4 (a) The triangle ABC , is such that $AB = 3$ cm, $BC = 5$ cm and $AC = 7$ cm.
The angle ABC is θ .
Find the value of θ .
- [3 marks]**
- 4 (b) Hence find the area of the triangle ABC . Give your answer in exact form.
- [3 marks]**
- 5 Given that $\sin x = 0.8$ find the values of $\sin(180^\circ - x)$ and $\sin(180^\circ + x)$.
- [2 marks]**

Trigonometry 1

Section B: A bit more thinking

- 6** A ship sails 5 km from S to T on a bearing of 062° and then 8 km from T to U on a bearing of 150° .
Calculate the distance SU giving your answer correct to two significant figures.
[3 marks]
- 7 (a)** Three points of an equilateral triangle lie on a circumference of length 10π .
Show that the area of the triangle is given by $\frac{75\sqrt{3}}{4} \text{ cm}^2$.
[5 marks]
- 7 (b)** Calculate the perimeter of the triangle, leave your answer in surd form.
[3 marks]
- 8 (a)** Consider a triangle ABC such that $\angle BAC$ is equal to 30° , $AB = x$, $BC = 4$ and $AC = 5$.
Show with your reasoning why $x^2 - 5\sqrt{3}x + 9 = 0$.
[2 marks]
- 8 (b)** Show that there are **two** solutions to the equation $x^2 - 5\sqrt{3}x + 9 = 0$ and interpret it in this context.
[2 marks]

Trigonometry 2

Section A: The basics

1 Simplify $\frac{\cos^2 \theta}{1 - \sin \theta}$

Circle your answer.

$1 - \sin \theta$

$1 + \cos \theta$

$1 + \sin \theta$

$1 - \cos \theta$

[1 mark]

2 Show that $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 \equiv 2$

[3 marks]

3 Solve $\sin x = -\cos x$ for $0^\circ \leq x \leq 180^\circ$

[2 marks]

4 Solve $\cos\left(\frac{x}{2} + 115^\circ\right) = -0.5$ for $0^\circ \leq x \leq 360^\circ$

[3 marks]

5 (a) Show that $\frac{\sin^2 \theta}{1 + \cos \theta} = 1 - \cos \theta$

[2 marks]

5 (b) Hence solve $\frac{\sin^2 \theta}{1 + \cos \theta} + 3 \cos^2 \theta = 3$ for $0^\circ \leq \theta \leq 360^\circ$, giving your answers to the nearest 0.1° .

[5 marks]

Trigonometry 2

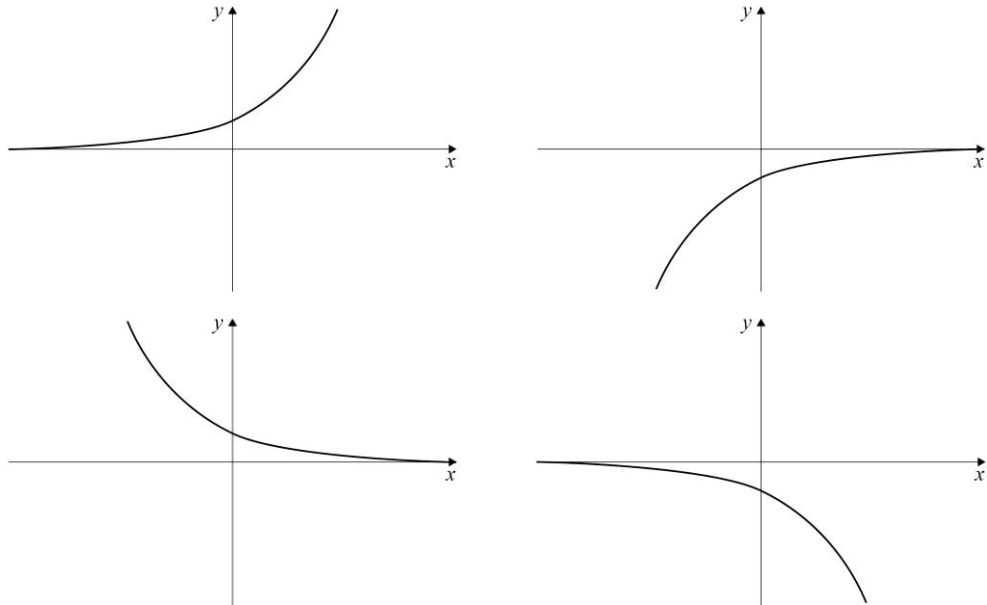
Section B: A bit more thinking

- 6 Solve $5 \sin x = 4 \cos x$ for $0^\circ \leq x \leq 360^\circ$, giving your answers to the nearest 0.1° .
[3 marks]
- 7 Solve $\sin(2x - 60^\circ) = 0.5$ for $0^\circ \leq x \leq 360^\circ$
[4 marks]
- 8 (a) Solve the simultaneous equations $k \sin x = 3$, $k \cos x = 7$ for $0^\circ \leq x \leq 360^\circ$, giving your answers to the nearest 0.1° .
[3 marks]
- 8 (b) State the coordinates of the maximum and minimum values of $f(x) = k \sin x$ for $0^\circ \leq x \leq 360^\circ$.
[2 marks]
- 9 Prove that $\frac{\tan \theta \sin \theta}{1 + \cos \theta} \equiv \frac{1}{\cos \theta} - 1$
[4 marks]

Exponentials 1

Section A: The basics

- 1 (a) Circle the correct graph for the function $y = a^{(x+3)}$ where a is a constant such that $a > 1$.



[1 mark]

- 1 (b) Sketch the graph $y = -b^x$ where b is a constant such that $0 < b < 1$.
State the coordinates of any intersections with the axes.

[2 marks]

- 2 Express $3 \log x + 0.5 \log y - 2 \log z$ as a single logarithm.

[3 marks]

- 3 Simplify $\ln e^x - 2 \ln e^{-x} + 0.5 \ln e^{2x}$

[2 marks]

- 4 Sketch the graph $y = \ln(ax)$ where a is a positive constant.
State the coordinates of any intersections with the axes.

[2 marks]

- 5 Express $\log \sqrt[3]{\frac{a^6}{b^3c}}$ in terms of $\log a$, $\log b$ and $\log c$.

[3 marks]

- 6 Express $\log_b 15 - 3 \log_b 3 - 2$ as a single logarithm.

[3 marks]

Exponentials 1

Section B: A bit more thinking

- 7** Given that $\log_{10} x = 550$, find the value of $\log_{10} \frac{1000}{\sqrt{x}}$ **[3 marks]**
- 8** The tangent to the graph $y = e^{kx}$ at $x = 3$ has the equation $e^b y + 4x = 13$.
Find the values of k and b . **[4 marks]**
- 9** Solve $\ln x = \ln(x + 6) - \ln(x + 2)$. **[5 marks]**
- 10** Find the exact coordinates of the intersection of the line $y = 4x + 7$ and the normal to the curve $y = e^{3x}$ at $x = 0$. **[4 marks]**

Section A: The basics

1 Solve $4^{2x+5} = 6$

Circle your answer.

$$x = \frac{1}{2} \log \frac{6}{4} - 5 \quad x = \frac{1}{2} (\log 6 - 5 \log 4) \quad x = \frac{1}{2} \left(\log \frac{6}{4} - 5 \right) \quad x = \frac{1}{2} \left(\frac{\log 6}{\log 4} - 5 \right)$$

[1 mark]

2 Solve $3^{x+2} = 0.4$ giving your answer to 3 significant figures.

[3 marks]

3 (a) The temperature T °C of a liquid at time t minutes is given by the equation

$$T = 50 + 35e^{-0.8t} \text{ for } t \geq 0$$

State the initial temperature of the liquid.

[1 mark]

3 (b) Calculate the rate of change of the temperature when $t = 2$ minutes.

[2 marks]

3 (c) Calculate to the nearest second the length of time for the temperature to decrease to 60°C.

[3 marks]

3 (d) The model represents a cup of tea cooling to room temperature. Explain why the prediction made by the model for large values of t may be inappropriate.

[2 marks]

4 (a) The table shows experimental values of x and y .

x	1	3	5	7
y	17.7	4.43	1.11	0.277

By plotting a suitable straight line graph, show that x and y are related by the equation $y = kb^x$ where k and b are constants to be found.

[3 marks]

4 (b) Hence find y when $x = 4$

[1 mark]

Section B: A bit more thinking

- 5 Two years ago £200 was deposited in a bank account accruing compound interest. This has now increased to £240.

Assuming no more money (other than interest) is added to or taken from the account, how much money will the account contain after a further 10 years?

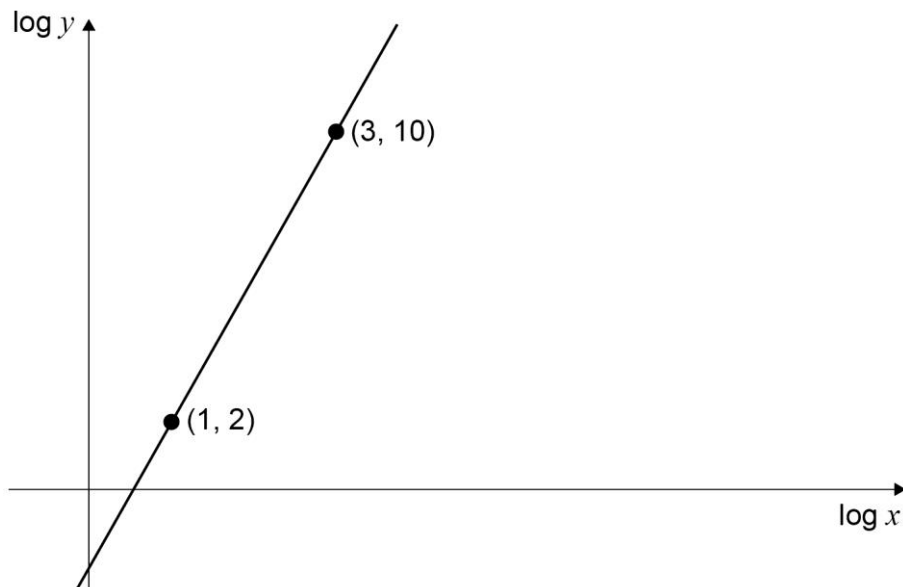
[5 marks]

- 6 The concentration of a particular drug in a patient's body decays exponentially. The initial dose given is 1.75 mg/ml and after five hours the concentration has dropped to 1.2 mg/ml.

Find to the nearest minute how much longer it will take for the concentration to drop to 0.8 mg/ml.

[6 marks]

7



Using the graph, express y in terms of x .

Hence find y when $x = 10$.

[5 marks]