

PiXL Independence:

Mathematics – Answer Booklet

KS5

Topic 1 – Basic Algebra

Contents:

- I. Basic skills check – 10 credits per skill check
- II. Short Exam Questions - 30 credits per section
- III. Further Practice – 30 credits each
- IV. Investigations – 80 credits each
- V. Academic stretch – 50 credits each

I. Basic Skills check

Answer the following questions. In order to improve your basic arithmetic you should attempt these without a calculator

Skills Check 1

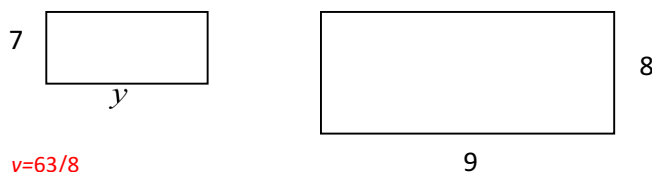
1. Write down the highest square factor of 50.

25

2. State whether true or false: $\sqrt{9 \times 4} = \sqrt{9} \times \sqrt{4}$

True

3. These two rectangles are similar. Find the value of y .



$y=63/8$

4. A solid cylinder has a base radius of r cm and a height of h cm. Write down a formula for the volume of the cylinder.

$V = \pi r^2 h$

5. Find the distance between the two points (3, 4) and (9, 12).

10

6. Solve the simultaneous equations $y = 2x + 1$ and $y = 7 - x$.

$x=2, y=5$

7. Solve the equation $6 - 3x = 2x + 1$

$x=1$

8. Expand the brackets $(2x - 1)(3x + 2)$

$6x^2 + x - 2$

9. Given $f(x) = x^2 + 5x - 2$ find the value of $f(4)$.

34

10. Solve $(x - 3)(2x + 5) = 0$.

$x=3, x=-\frac{5}{2}$

Skills Check 2

1. Write down two numbers that have a product of 24 and a sum of 11.

7 and 3

2. State whether true or false: $\sqrt{100 \times 4} = \sqrt{400}$

True

3. Factorise $x^2 - 2x - 35$.

$(x-7)(x+5)$

4. A solid cylinder has a base radius of r cm and a height of h cm. Write down a formula for the surface area of the cylinder.

$A = 2\pi r^2 + 2\pi rh$

5. Find the distance between the two points $(-1, -2)$ and $(4, 10)$.

13

6. Solve $x^2 + 15x + 56 = 0$.

$x=-8, x=-7$

7. Simplify $(\sqrt{2} + 1)(\sqrt{2} - 3)$.

$-(1+2\sqrt{2})$

8. Solve $3x(x-3) = -4x-2$

$x=1, x=\frac{2}{3}$

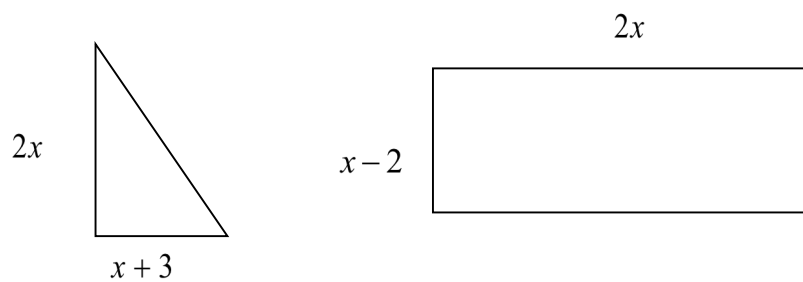
9. Solve the simultaneous equations

$$3x - 4y = 4$$

$$5x + 4y = 28$$

$x=4, y=2$

10. The triangle and the rectangle have the same area. Find the value of x .



$x=7$

Skills check 3

1. Write down the reciprocal of 2.

0.5

2. Write the equation of the line $\frac{3}{4}x - 2 = \frac{1}{6}y$ in the form $ax + by + c = 0$.

$$\frac{3}{4}x - \frac{1}{6}y - 2 = 0$$

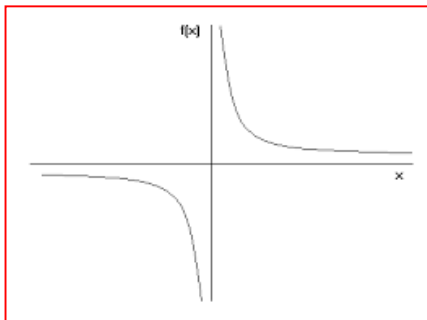
3. Express $\frac{6}{\sqrt{3}}$ in the form $k\sqrt{3}$.

$$2\sqrt{3}$$

4. Factorise $x^2 + 7x$.

$$x(x+7)$$

5. Sketch the graph of $y = \frac{1}{x}$.



6. Solve the simultaneous equations.

$$2y + 3x = 18$$

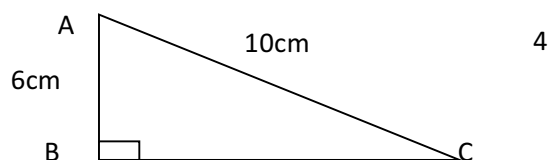
$$5y - x = 11$$

$$x=4, y=3$$

7. Solve the inequality $3x - 4 > 2$.

$$x > 2$$

8. In the given right-angled triangle ABC, find the length of BC.



BC=8cm

9. Given $P(x) = x^3 + 4x^2 + x - 6$ find the value of $P(1)$.

0

10. Write down the mid-point of the line joining (-2, -5) and (7, -3).

$(\frac{5}{2}, -4)$

II. Short Exam Questions

Section 1- Quadratic graphs. ?? credits for completing and marking this section

1. Here are the equations of nine parabolas.

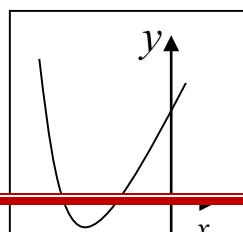
- i.** $y = (x-3)(x-8)$
- ii.** $y = 14 + 5x - x^2$
- iii.** $y = 6x^2 - x - 70$
- iv.** $y = x(4-x)$
- v.** $y = (x+2)(x-8)$
- vi.** $y = x^2 + 9x + 20$
- vii.** $y = x^2 - 36$
- viii.** $y = -3(x+3)(x+7)$
- ix.** $y = x^2 + 4x + 4$

Answer the following questions, the answer can be 'none'

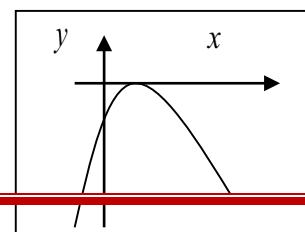
- A. Which of the parabolas cross the y-axis at a positive value of y? **1,2,6,9**
- B. For which of the parabolas is the vertex at the highest point of the graph? **2,4,8**
- C. For which of the parabolas is the vertex to the left of the y-axis? **6,8,9**
- D. Which of the parabolas pass through the origin? **4**
- E. Which of the parabolas does not cross the x-axis at two separate points? **9**
- F. Which of the parabolas has the y-axis as its axis of symmetry? **7**
- G. Which two of the parabolas have the same axis of symmetry? **None**
- H. Which of the parabolas have the vertex in the fourth quadrant? **1,5**

1. Suggest a possible equation for each of the following graphs:

i.



ii.



Any with $(x+a)(x+b)$

eg $-(x-3)(x-3)$

eg $-(x-5)(x-7)$

eg $2x^2 + 8x + 10$

b coefficient needs to be bigger than the a

iii.

iv.

ection 2- Completing the square and the Discriminant

- By completing the square, solve the following quadratic equations, giving your answers in surd form where appropriate.
 - $x^2 - 10x - 24 = 0$ $x=12$ or $x=-2$
 - $x^2 - 4x - 1 = 0$ $x = 2 \pm \sqrt{5}$
 - $10x^2 + 60x + 40 = 0$ $x = -3 \pm \sqrt{5}$
- By completing the square, find in terms of k the roots of the equation $x^2 + 2kx - 7 = 0$. Prove that, for all real values of k , the roots of $x^2 + 2kx - 7 = 0$ are real and different. Given that $k = \sqrt{2}$, find the exact roots of the equation. $x = -k \pm \sqrt{k^2 + 7}$. Exact roots are $-\sqrt{2} \pm 3$
- By first completing the square, explain why the equation $x^2 + 10x + 27 = 0$ has no real roots. $(x + 5)^2 + 2 = 0$ when this is rearranged we would need to do the calculation $\sqrt{-2}$ which we are not able to
- Show that $f(x) = 3x^2 - 9x + 1$ can be written as $f(x) = 3\left(x - \frac{3}{2}\right)^2 - \frac{23}{4}$. What is the minimum value of $f(x)$ and what is the value of x that makes $f(x)$ a minimum?
Minimum value is $-\frac{23}{4}$ this happens when $x = \frac{3}{2}$

Section 3: Inequalities and Graphs

1. Given that the equation $2x^2 + 4px + q = 0$ has no real roots, show that $2p^2 < q$. If there are no real roots, then $b^2 - 4ac < 0$. So $(4p)^2 - 4 \times 2 \times q < 0$.

$$\begin{aligned} 16p^2 - 8q &< 0 \\ 2p^2 - q &< 0 \end{aligned}$$

2. The quadratic equation $x^2 + 2kx + (k + 56) = 0$ has repeated roots.

a) Show that $k^2 - k - 56 = 0$. If there are repeated roots, then $b^2 - 4ac = 0 \dots$

$$\text{so } (2k)^2 - (4 \times 1 \times (k + 56)) = 0$$

$$4k^2 - 4k - 224 = 0$$

$$k^2 - k - 56 = 0 \text{ as required}$$

b) Hence find the two possible values for k. $k = -7, k = 8$

3. Given that the quadratic equation $x^2 + (2m + 3)x + m^2 = 0$ has real roots, show that $m \geq -\frac{3}{4}$.

If real roots the $b^2 - 4ac > 0$

$$(2m + 3)^2 - 4m^2 > 0$$

Expand and simplify gives $12m + 9 > 0$ so $m > -\frac{3}{4}$ as reqd

4. Use the sketch shown opposite to write down the solutions to these inequalities:

a) $x^3 - 3x^2 + 4 < 0$

$$x < -1$$

b) $x^3 - 3x^2 + 4 > 0$

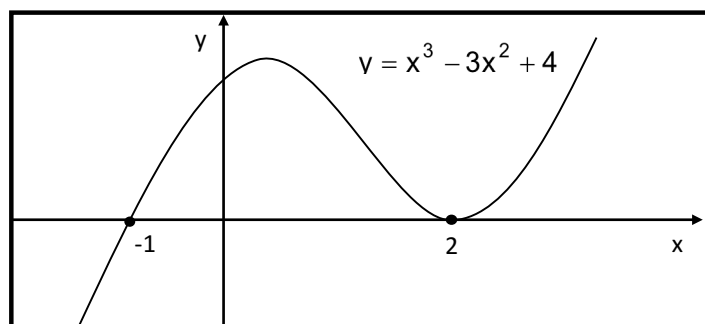
$$x > -1$$

c) $x^3 - 3x^2 + 4 \leq 0$

$$x \leq -1$$

d) $x^3 - 3x^2 + 4 \geq 0$

$$x \geq -1$$



5. Use the graph shown opposite to write down the solutions to these inequalities:

(i) $(x + 1)(x - 2) < 0$

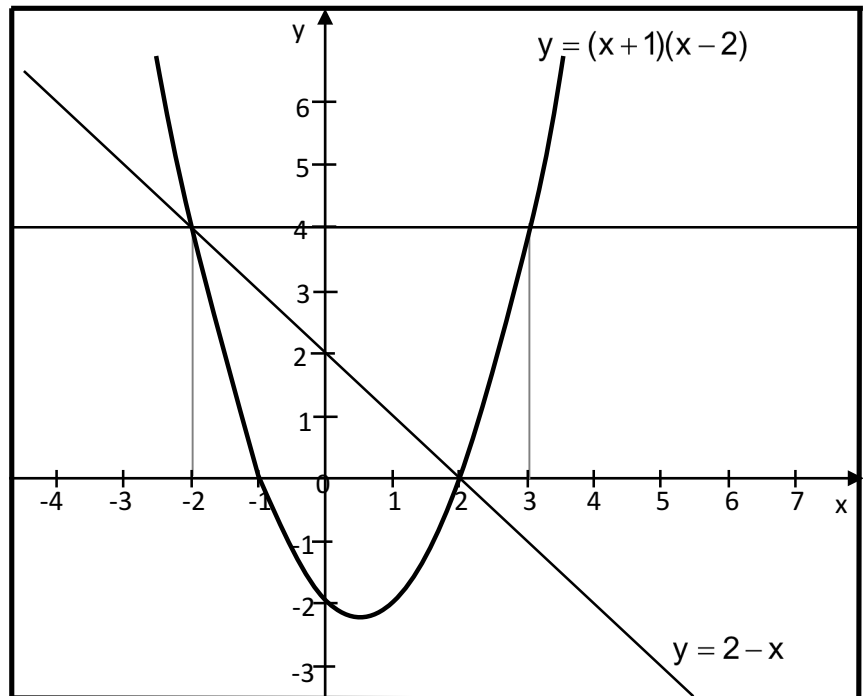
$-1 < x < 2$

(ii) $(x + 1)(x - 2) > 4$

$x < -2$ and $x > 3$

(iii) $(x + 1)(x - 2) \leq 2 - x$

$-2 \leq x \leq 2$



Section 4: Mixed Questions

- 1) The straight line $y - x = 2$ intersects the circle $x^2 + y^2 + 5x = 9$ at the points A and B. Find the coordinates of A and B.

Use substitution to form quadratic equation to solve. Gives coordinates (0.5, 2.5) (-5, -3)

- 2) Write down

a) A rational number which lies between 4 and 5.

b) An irrational number which lies between 4 and 5.

c) A student says, 'When you multiply two irrational numbers together the answer is always an irrational number'.

Simplify $(2 + \sqrt{3})(2 - \sqrt{3})$ and comment on the student's statement. When you expand you get an answer of 1, so the student's statement is not correct

- 3)

a) Express $x^2 + 12x + 11$ in the form $(x + a)^2 + b$, finding the values of the constants a and b . $a=6$
 $b= -25$

b) State the minimum value of the expression $x^2 + 12x + 11$.

Minimum value is at (-6, -25)

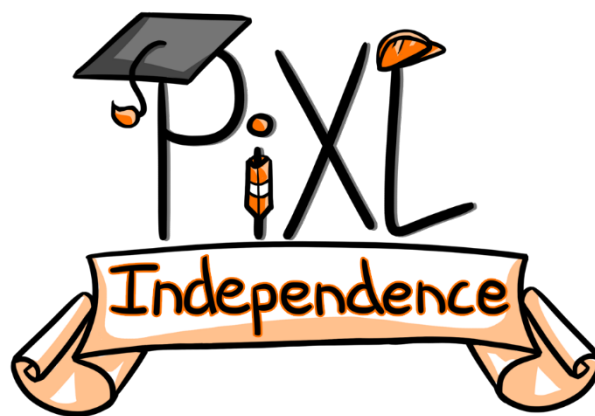
c) Determine the values of k for which the quadratic equation $x^2 + 3(k - 2)x + (k + 5) = 0$ has equal roots. Use the discriminant

$x=4$ or $x=4/9$

- 4)

a) Express $x^2 + 4x - 5$ in the form $(x + a)^2 + b$, finding the values of the constants a and b . $a=2$ $b= -9$

b) Hence, or otherwise, find the range of values for which $x^2 + 4x - 5 > 0$.



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