PiXL Independence:

Mathematics - ANSWERS

KS5

Topic 2 – Polynomials

I. Basic skills check – 10 credits per skill check

II. Short Exam Questions - 30 credits per section

III. Further Practice – 30 credits each

IV. Investigations – 80 credits each

V. Academic stretch – 50 credits each

I. Basic Skills check

Answer the following questions. In order to improve your basic arithmetic you should attempt these without a calculator

10 credits for completing this quiz.

Skills check 1

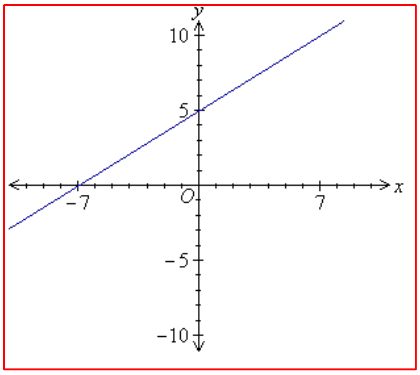
1. Rewrite the equation of the straight line $2x + 3y = 6$ in the form $y = mx + c$.

   $y = -\frac{2}{3} + 2$

2. Factorise $x^2 + 5x - 24$.

   (x+8)(x-3)

3. Sketch the graph of $y = x + 5$.



4. Write $\frac{1}{x^3}$ in the form x^n.

   x^{-3}

5. Show that the lines $2x - 5y = 10$ and $10y - 4x - 5 = 0$ are parallel.

   By rearranging both equations into the form y=mx+c we get,

   2x-5y=10 $\Rightarrow$ $y = \frac{2}{5}x - 2$

   10y-4x-5=0 $\Rightarrow$ $y = \frac{2}{5}x + \frac{1}{2}$

   As both the gradients (m-values) equal $\frac{2}{5}$ we can conclude that the two lines are parallel.

6. Given $f(x) = x^3 + 3x^2 - 6x - 8$, find the value of $f(2)$.

   0

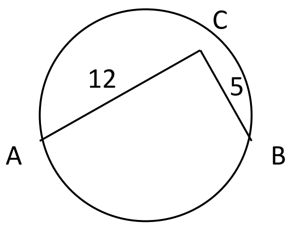
7. Express $\left(\sqrt{7}+1\right)^2$ in the form $a + b\sqrt{7}$, where a and b are integers.

   8+2√7

8. Solve the inequality $5 \langle 2x - 1 \langle 17$.

   x < 3 < 9

9. Find the length of AB, given X is the centre of the circle.



   AB = 13

<u>Skills check 2</u>

1. Solve the inequality $6(x+3) \rangle 8 - 2(x+1)$.

   x > - $\frac{3}{2}$

2. Sketch the graph of $y = 3x^4$.

3. Work out the point of intersection of the two lines $x - 2y = 5$ and $2x = 5y + 7$.

   x = 11, y = 3

4. Simplify $6\sqrt{2} + 5\sqrt{8}$.

   16√2

5. Write down the mid-point of (4, 5) and (6, 3).

   (5, 4)

6. Write $\frac{1}{4x^3}$ in the form kx^n.

$4x^{-3}$

7. Solve the equation $2x^2 + x - 4 = 0$, leaving your answer in surd form.

x = $\frac{1}{4}(-1-\sqrt{33})$, x = $\frac{1}{4}(\sqrt{33} - 1)$

8. It is given that $f(x) = x^3 + 7x^2 + 8x + 10$. Find the value of $f(1)$ and $f(-1)$.

$f(1) = 26$, $f(-1) = 8$

9. The points A and B have coordinates (12, 5) and (7, 3). Find the gradient of AB.

Gradient = $\frac{2}{5}$

10. Factorise $x^2 - 9$.

(x+3)(x-3)

Skills check 3

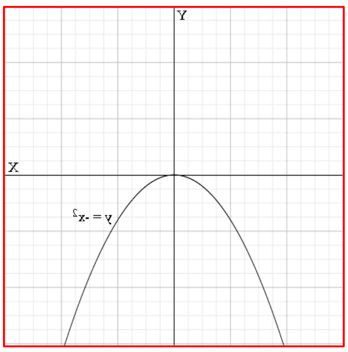
1. Write down the reciprocal of $\frac{1}{5}$.

5

2. Given $\frac{3+\sqrt{5}}{4+\sqrt{5}} = p + q\sqrt{5}$, where p and q are rational numbers, find p and q.

$\frac{7}{11} + \frac{1}{11}\sqrt{5}$

3. Sketch the graph of $y = -x^2$.



4. Solve $6x^2 + 11x + 3 = 0$ by factorisation.

x = $\frac{-3}{2}$, x = $\frac{-1}{3}$

5. Solve the inequality $-3 \leq \frac{x}{2} \leq 5$.

-6 < x < 10

6. Given $P(x) = 2x^3 + x^2 - 4x + 5$, evaluate $P(2)$.

17

7. Write down the mid-point of (2, 10) and (-3, 0).

(-0.5, 5)

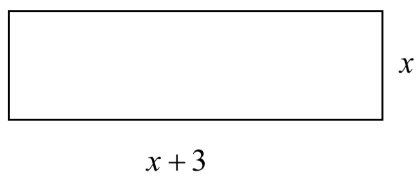
8. Write down the gradient of the line joining (2, 10) and (-3, 0).

m = 2

9. Solve the simultaneous equations $3x + 2y = 6$ and $y = 5x - 10$.

x = 2, y=0

10.



The area of this rectangle is y cm^2 and the perimeter is y cm. Find the dimensions of the rectangle.

x=3. Therefore, length = 6 and width = 3.

II. Short Exam Questions

Section 1- Algebraic division

1. Given that $4x^3 - 25x^2 - 23x + 14 \equiv (x-7)(px^2 + qx + r)$, find the values of the constants p, q and r.
P=4 q=3 r= -2

2. You are given that $f(x) = x^3 + x^2 - 14x - 24$,

a. Write $f(x)$ in the form $(x+2)(ax^2+bx+c)$. $(x+2)(x^2-x-12)$

b. By first factorising the quadratic part of your answer to (a), express $f(x)$ as a product of three linear factors. $(x-4)(x+2)(x+3)$

3. You are given that $g(x)=x^3-4x^2-7x+10$,

a. Write $g(x)$ in the form $(x-1)(ax^2+bx+c)$. $(x-1)(x^2-3x-10)$

b. Hence express $g(x)$ as a product of three linear factors. $(x-5)(x+2)(x-1)$

c. Hence solve the equation $g(x)=0$. $x=5, x=-2, x=1$

4. The polynomial $p(x)$ is defined by $p(x)=3x^3-29x^2+62x+24$

You are given that $(x-6)$ is a factor of $p(x)$.

a) Factorise $p(x)$ completely. $\mathbf{(3x+1)(x-6)(x-4)}$

b) Hence simplify $\dfrac{2x^2-8x}{3x^3-29x^2+62x+24}$. $\dfrac{2x}{(3x+1)(x-6)}$

5. The polynomial $p(x)$ is defined by $p(x)=2x^3+5x^2+x-2$.

Given that $(2x-1)$ is a factor of $p(x)$

a. Write $p(x)$ as a product of three linear factors with integer coefficients. $\mathbf{(2x-1)(x+2)(x+1)}$

b. Simplify the algebraic fraction $\dfrac{3x^2+6x}{2x^3+5x^2+x-2}$ as far as possible. $\dfrac{3x}{(2x-1)(x+1)}$

Section 2-Factor Theorem

1. Given that $f(x)=x^3-7x-6$,

a) find f(1), f(-1), f(2), f(-2), f(3) and f(-3). f(1)= -12, f(-1)=0 , f(2)=-12, f(-2)=0, f(3)=0

b) hence write $f(x)$ as a product of three linear factors. $(x+2)(x+1)(x-3)$

c) solve the equation $f(x)=0$. $x=-2, x=-1, x=3$

2. Given that $g(x)=x^3-3x^2-6x+8$,

a) use the factor theorem to show that (x-1) is a factor of g(x). f(1)=0

b) factorise g(x) completely. $(x+2)(x-1)(x-4)$

c) solve the equation g(x)=0. $x=-2, x=1, x=-4$

3. Given that $h(x)=x^3-3x^2-16x-12$,

a) use the factor theorem to show that (x+2) is a factor of h(x). f(-2) =0

b) write h(x) in the form $(x+2)(x^2+px+q)$ where p and q are constants to be determined.

$(x+2)(x^2-5x-6)$

c) solve the equation h(x)=0, leaving your answers in surd form where appropriate.

$x=-2, x=6, x=-1$

4. Given that $g(x)=x^3+ax+6$,

a) If (x+3) is a factor of g(x), show that a=-7. When f(-3)=0 gives a=-7

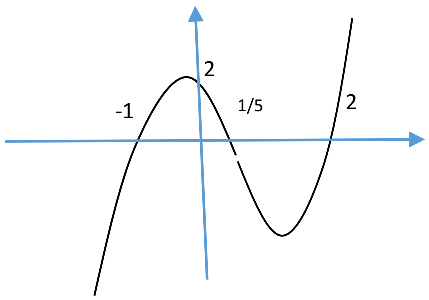
b) Hence solve the equation g(x)=0, giving answers in **surd form** where appropriate. X=-3 x=1 x=2

5. The function $f(x)=x^3+Ax^2+Bx+10$ has factors (x+2) and (x-5).

a) Use this information to form and solve two simultaneous equations to find A and B. A=-4 B=-7
b) Factorise f(x) completely. (x+2)(x-5)(x-1)

Section 3- Polynomials and graphs

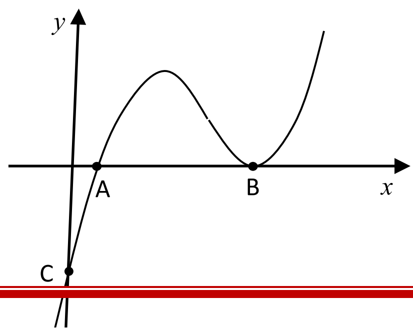
1. Given that $f(x)=5x^3-6x^2-9x+2$,
   a. Use the factor theorem to show that $(x-2)$ is a factor of $f(x)$. f(2)=0
   b. Solve the equation $f(x)=0$. x=2, x=-1 , x=1/5
   c. Hence sketch the graph $y=f(x)$, labelling the points where the curve crosses the coordinate axes.



2. You are given that $f(x)=(x+1)(x+2)(x-2)$
   a. Solve the equation $f(x)=0$. X=-1, x=-2, x= 2
   b. What does this tell you about the graph of $y=f(x)$? This tells us where the graph will cross the x-axis
   c. Where will the graph cross the y-axis ? when x=0 y=-4 so (0, -4)
   d. Sketch the graph of $y=f(x)$.
   e. Write the equation of the graph in the form $y=ax^3+bx^2+cx+d$. x^3+x^2-4x-4

3. Solve the equation of $(x+4)(x-2)(x-1)=0$. X=-4, x=2, x=1
   a. What does this tell you about the graph of $y=(x+3)(x+2)(x-1)$? This tells us where the graph will cross the x- axis
   b. Where will the graph cross the y-axis? When x=0 y=8 so (0,8)
   c. Sketch the graph of $y=(x+4)(x+2)(x-1)$.
   d. Write the equation of the graph in the form $y=ax^3+bx^2+cx+d$.
   x^3+3x^2-6x-8

3. The sketch opposite shows the curve $y=(2x-3)(x-5)^2$.



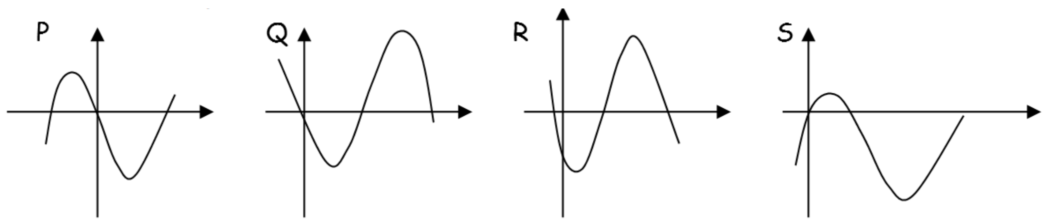
   a. The constants A, B and C on the sketch indicate the points where the curve meets the coordinate axes.

Write down the values of A, B and C. A=$\frac{3}{2}$ B=5 C=-75

b. Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.

$2x^3 - 23x^2 + 80x - 75$

4. Show that $x(x-4)(3-x)$ which of the graphs below is a sketch of $y = -x^3 + 7x^2 - 12x$.



Expand the brackets to give $(x^2 - 4x)(3 - x)$

$$= 3x^2 - 12x - x^3 + 4x^2$$

$$= -x^3 + 7x^2 - 12x$$

Graph Q

Section 4- Mixed questions.

1. $f(x) = x^3 + (a+1)x^2 - 18x + b$, where a and b are integers.

Given that $(x - 4)$ is a factor of $f(x)$,

(a) show that $16a + b + 8 = 0$. $f(4) = 64 + 16(\mathrm{a}+1) - 72 + \mathrm{b} = 0$

$$-8 + 16\mathrm{a} + 16 + \mathrm{b} = 0$$

$$16\mathrm{a} + 8 + \mathrm{b} = 0 \text{ as reqd.}$$

Given that $(x + a)$ is also a factor of $f(x)$, and that $a > 0$,

(b) show that $a^2 + 18a + b = 0$. $f(-a) = (-a)^3 + (-a+1)a^2 + 18a + b$

$$a^2 + 18a + b = 0$$

(c) Hence find the value of a and the corresponding value of b. Use sim equations with the substitution b=-16a-8. Gives a=-4 or a=2 BUT can only have a>0 so a=2. b=-40

(d) Factorise $f(x)$ completely. $f(x) = (x-4)(x+2)(x+5)$

2. The polynomial $P(x) = x^3 - 4x^2 + kx - 4$ leaves a remainder of –2 when divided by $(x-1)$.

a) Find the value of the constant k . K=5

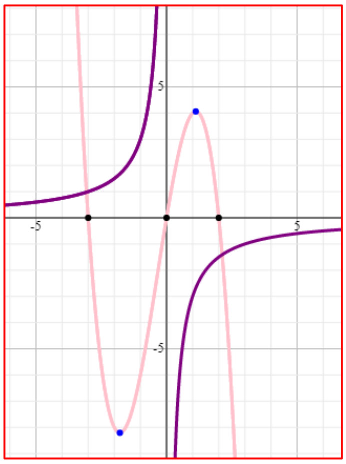
b) Show that $(x-2)$ is **not** a factor of $P(x)$. $f(2) = 8 - 16 + 10 - 4 = -2$

Therefore not a factor.

3. Sketch on a single diagram the following graphs

a) $y = x(x+3)(2-x)$. Crosses at (-3,0)(0,0)(2,0)

b) $y = -\frac{3}{x}$



c) Using your sketch, giving reasons, find the number of real solutions to the equation

$x(x+3)(2-x) + \frac{3}{x} = 0$

2 Solutions

4. The function The function $f(x) = x^3 + Ax^2 + Bx - 30$ has factors (x-2) and (x+5).

a) Use this information to form and solve two simultaneous equations to find A and B.

$f(2) gives\ equation \quad 4A + 2B = 22$

$f(-5) gives\ equation\ 25A - 5B = 155$

Solving gives A=6 B=-1

b) Factorise f(x) completely. $(x+3)(x-2)(x+5)$

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