

PiXL Independence:

Mathematics - Student Booklet

KS5

Topic 2 – Polynomials

Contents:

- I. Basic skills check – 10 credits per skill check
- II. Short Exam Questions - 30 credits per section
- III. Further Practice – 30 credits each
- IV. Investigations – 80 credits each
- V. Academic stretch – 50 credits each

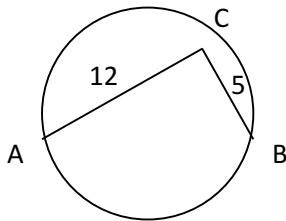
I. Basic Skills Check

Answer the following questions. In order to improve your basic arithmetic you should attempt these without a calculator

10 credits for completing this quiz.

Skills check 1

1. Rewrite the equation of the straight line $2x + 3y = 6$ in the form $y = mx + c$.
2. Factorise $x^2 + 5x - 24$.
3. Sketch the graph of $y = x + 5$.
4. Write $\frac{1}{x^3}$ in the form x^n .
5. Show that the lines $2x - 5y = 10$ and $10y - 4x - 5 = 0$ are parallel.
6. Given $f(x) = x^3 + 3x^2 - 6x - 8$, find the value of $f(2)$.
7. Express $(\sqrt{7} + 1)^2$ in the form $a + b\sqrt{7}$, where a and b are integers.
8. Solve the inequality $5 < 2x - 1 < 17$.
9. Find the length of AB, given X is the centre of the circle.



Skills check 2

1. Solve the inequality $6(x+3) > 8 - 2(x+1)$.
2. Sketch the graph of $y = 3x^4$.
3. Work out the point of intersection of the two lines $x - 2y = 5$ and $2x = 5y + 7$.
4. Simplify $6\sqrt{2} + 5\sqrt{8}$.
5. Write down the mid-point of $(4, 5)$ and $(6, 3)$.
6. Write $\frac{1}{4x^3}$ in the form kx^n .
7. Solve the equation $2x^2 + x - 4 = 0$, leaving your answer in surd form.
8. It is given that $f(x) = x^3 + 7x^2 + 8x + 10$. Find the value of $f(1)$ and $f(-1)$.
9. The points A and B have coordinates $(12, 5)$ and $(7, 3)$. Find the gradient of AB.
10. Factorise $x^2 - 9$.

Skills check 3

1. Write down the reciprocal of $\frac{1}{5}$.
2. Given $\frac{3+\sqrt{5}}{4+\sqrt{5}} = p + q\sqrt{5}$, where p and q are rational numbers, find p and q .
3. Sketch the graph of $y = -x^2$.
4. Solve $6x^2 + 11x + 3 = 0$ by factorisation.
5. Solve the inequality $-3 \leq \frac{x}{2} \leq 5$.
6. Given $P(x) = 2x^3 + x^2 - 4x + 5$, evaluate $P(2)$.
7. Write down the mid-point of $(2, 10)$ and $(-3, 0)$.
8. Write down the gradient of the line joining $(2, 10)$ and $(-3, 0)$.
9. Solve the simultaneous equations $3x + 2y = 6$ and $y = 5x - 10$.
- 10.



$x + 3$

The area of this rectangle is $y \text{ cm}^2$ and the perimeter is $y \text{ cm}$. Find the dimensions of the rectangle.

II. Short Exam Questions

Section 1- Algebraic division

- Given that $4x^3 - 25x^2 - 23x + 14 \equiv (x - 7)(px^2 + qx + r)$, find the values of the constants p , q and r .
- You are given that $f(x) = x^3 + x^2 - 14x - 24$,
 - Write $f(x)$ in the form $(x + 2)(ax^2 + bx + c)$.
 - By first factorising the quadratic part of your answer to (a), express $f(x)$ as a product of three linear factors.
- You are given that $g(x) = x^3 - 4x^2 - 7x + 10$,
 - Write $g(x)$ in the form $(x - 1)(ax^2 + bx + c)$.
 - Hence express $g(x)$ as a product of three linear factors.
 - Hence solve the equation $g(x) = 0$.
- The polynomial $p(x)$ is defined by $p(x) = 3x^3 - 29x^2 + 62x + 24$
You are given that $(x - 6)$ is a factor of $p(x)$.
 - Factorise $p(x)$ completely.
 - Hence simplify $\frac{2x^2 - 8x}{3x^3 - 29x^2 + 62x + 24}$.
- The polynomial $p(x)$ is defined by $p(x) = 2x^3 + 5x^2 + x - 2$.
Given that $(2x - 1)$ is a factor of $p(x)$
 - Write $p(x)$ as a product of three linear factors with integer coefficients.
 - Simplify the algebraic fraction $\frac{3x^2 + 6x}{2x^3 + 5x^2 + x - 2}$ as far as possible.

Section 2-Factor Theorem

1. Given that $f(x) = x^3 - 7x - 6$,
 - a) find $f(1)$, $f(-1)$, $f(2)$, $f(-2)$, $f(3)$ and $f(-3)$,
 - b) hence write $f(x)$ as a product of three linear factors,
 - c) solve the equation $f(x) = 0$.

2. Given that $g(x) = x^3 - 3x^2 - 6x + 8$,
 - a) use the factor theorem to show that $(x-1)$ is a factor of $g(x)$,
 - b) factorise $g(x)$ completely,
 - c) solve the equation $g(x)=0$.

3. Given that $h(x) = x^3 - 3x^2 - 16x - 12$,
 - a) use the factor theorem to show that $(x+2)$ is a factor of $h(x)$,
 - b) write $h(x)$ in the form $(x + 2)(x^2 + px + q)$ where p and q are constants to be determined.
 - c) solve the equation $h(x)=0$, leaving your answers in surd form where appropriate.

4. Given that $g(x) = x^3 + ax + 6$,
 - a) If $(x+3)$ is a factor of $g(x)$, show that $a=-7$.
 - b) Hence solve the equation $g(x)=0$, giving answers in **surd form** where appropriate.

Section 3- Polynomials and graphs

1. Given that $f(x) = 5x^3 - 6x^2 - 9x + 2$,
 - a. Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.
 - b. Solve the equation $f(x) = 0$.
 - c. Hence sketch the graph $y = f(x)$, labelling the points where the curve crosses the coordinate axes.

2. You are given that $f(x) = (x + 1)(x + 2)(x - 2)$
 - a. Solve the equation $f(x) = 0$.
 - b. What does this tell you about the graph of $y = f(x)$?
 - c. Where will the graph cross the y-axis?
 - d. Sketch the graph of $y = f(x)$.
 - e. Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.

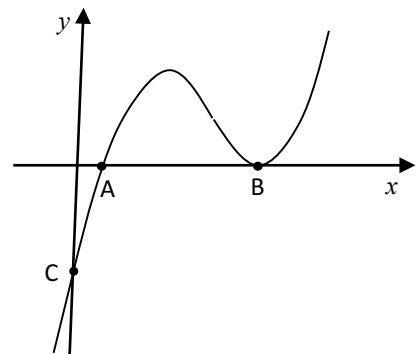
3. Solve the equation of $(x + 4)(x - 2)(x - 1) = 0$.
 - a. What does this tell you about the graph of $y = (x + 3)(x + 2)(x - 1)$?
 - b. Where will the graph cross the y-axis?
 - c. Sketch the graph of $y = (x + 4)(x + 2)(x - 1)$.
 - d. Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.

3. The sketch opposite shows the curve $y = (2x - 3)(x - 5)^2$.

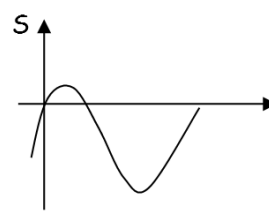
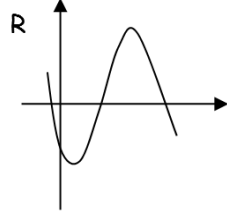
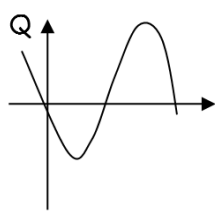
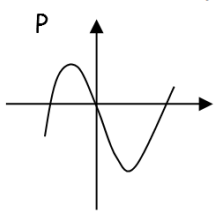
- a. The constants A, B and C on the sketch indicate the points where the curve meets the coordinate axes.

Write down the values of A, B and C.

- b. Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.



4. Show that $x(x-4)(3-x)$ is equivalent to $-x^3 + 7x^2 - 12x$. Which of the graphs below is a sketch of $y = -x^3 + 7x^2 - 12x$.



Section 4- Mixed questions.

1. $f(x) = x^3 + (a + 1)x^2 - 18x + b$, where a and b are integers.

Given that $(x - 4)$ is a factor of $f(x)$,

(a) show that $16a + b + 8 = 0$

Given that $(x + a)$ is also a factor of $f(x)$, and that $a > 0$,

(b) show that $a^2 + 18a + b = 0$.

(c) Hence find the value of a and the corresponding value of b .

(d) Factorise $f(x)$ completely.

2. The polynomial $P(x) = x^3 - 4x^2 + kx - 4$ leaves a remainder of -2 when divided by $(x - 1)$.

a) Find the value of the constant k .

b) Show that $(x - 2)$ is **not** a factor of $P(x)$.

3. Sketch on a single diagram the following graphs

a) $y = x(x + 3)(2 - x)$

b) $y = -\frac{3}{x}$

c) Using your sketch, giving reasons, find the number of real solutions to the equation

$$x(x + 3)(2 - x) + \frac{3}{x} = 0$$

4. The function $f(x) = x^3 + Ax^2 + Bx - 30$ has factors $(x - 2)$ and $(x + 5)$.

a) Use this information to form and solve two simultaneous equations to find A and B .

b) Factorise $f(x)$ completely.

III. Further Practice

- 1. Watch the video and answer all the questions. Try the interactive resource at the bottom of the video. What do you notice about the graphs?**
<https://www.examsolutions.net/tutorials/sketching-quadratic-graphs/?level=A-Level&board=Edexcel&module=C1&topic=1196>
- 2. Answer the questions and check your solutions**
http://www.mesacc.edu/~scotz47781/mat120/notes/divide_poly/synthetic/synthetic_division_practice.html
- 3. Choose at least two of the sections in this unit to read and make notes on.**
<http://www.coolmath.com/algebra/22-graphing-polynomials>
- 4. Assignment task to complete**
<http://tutorial.math.lamar.edu/ProblemsNS/Alg/Polynomials.aspx>
- 5. Create your own worksheet with at least 10 polynomial multiplication problems and answer them**
<http://www.webmath.com/wsheel1.html>
- 6. [http://furthermaths.org.uk/manager_area/files/Problem%20solving%20series%2012.03Q Quadratics and Cubics 3rd edition.pdf](http://furthermaths.org.uk/manager_area/files/Problem%20solving%20series%2012.03Q%20Quadratics%20and%20Cubics%203rd%20edition.pdf)**

IV. Investigations

For each of the following you should carry out the investigations then read the notes. You need to keep a detailed summary of what methods/ approaches you have tried and what you then changed each time.

- 1. Read the notes on the page and carry out the graph investigation. Complete the worksheet included, there's also an extension task to gain extra credit**
<https://www.teachmathematics.net/page/14038/exponential-graphs>
- 2. Euler's 9 Point Circle.** Read the article and follow the instructions in order to create a 9 point circle. Create a document/ poster to instruct someone else on how to do this.
<https://ibmathsresources.com/2017/09/30/eulers-9-point-circle/>
- 3. Read the article, follow some of the links. Can you write a summary of the key points and the maths involved.**
<https://nrich.maths.org/2769>
- 4. Read the article and the comments. Which arguments are convincing? Are there flaws in the responses? What would your response be if you were to comment on the article?**
<https://plus.maths.org/content/maths-minute-two-envelopes-problem>
- 5. Risp**
<http://www.s253053503.websitehome.co.uk/risps/risp-8.pdf>

V. Academic Reading

1. Pick one subject from the list and follow the materials and questions to extend your understanding about mathematics in other subjects

<https://integralmaths.org/course/view.php?id=166>

2. Read the following lecture notes and make a detailed summary

<https://www.maths.ox.ac.uk/system/files/attachments/lecture2.pdf>

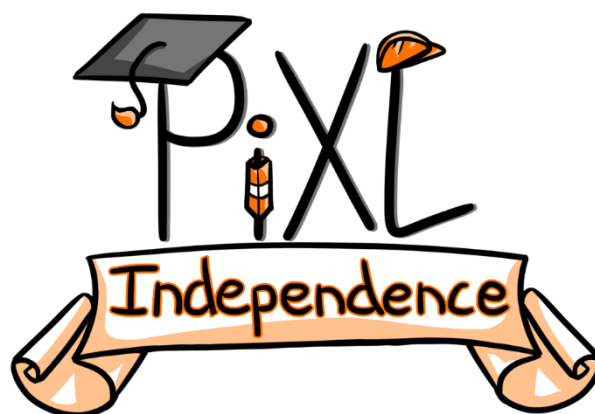
3. Read the history of algebra article and make a detailed summary of the key points. Follow at least one of the links to extend your reading.

[http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Fund theorem of algebra.html](http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Fund_theorem_of_algebra.html)

4. Complete STEP assignment 1/ read the article to extend your understanding

<https://maths.org/step/assignments/assignment-1>

<https://plus.maths.org/content/ringing-changes>



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