# PiXL Independence:

Mathematics - Answers

KS5

Topic 2 – Co-ordinate Geometry

I. Basic Skills check

Answer the following questions. In order to improve your basic arithmetic you should attempt these without a calculator

<u>Skills check 1.</u>

1. Sketch the graph of $y = \sqrt{x}$.

2. Write down the reciprocal of $-\dfrac{1}{10}$. **-10**

3. Describe the geometrical transformation which maps the parabola $y = x^2$ to $y = x^2 + 3$.

   Translation in vector $\binom{0}{3}$

4. Solve $(2x - 3)^2 = 36$.   **$x = \frac{3 \pm 6}{2}$**

5. Write $-\dfrac{1}{5\sqrt{x}}$ in the form kx^n. **$\frac{1}{5}x^{-\frac{1}{2}}$**

6. Solve the inequality $4(x - 2) \ge 7 - 2(x - 3)$. **$x \ge \frac{21}{6}$**

7. State where the curve $y = (2x - 1)(x + 7)$ crosses the x-axis. **When $x = \frac{1}{2}$ or $x = -7$**

8. Small trays are to be made from rectangular pieces of card. Each piece of card is 8 cm by 5 cm and the tray is formed by removing squares of side x cm from each corner and folding the remaining card along the dotted lines, as shown in the diagram, to form an open-topped box. Write down a formula for the volume of this box.
   $4x^3 - 26x^2 + 40x$
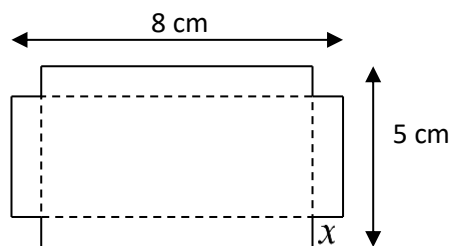
9. Express $\sqrt{8}(7 - \sqrt{2})$ in the form $a + b\sqrt{2}$. **$7\sqrt{8} - 4 = 14\sqrt{2} - 4$**

10. Write $x^2 + 10x + 30$ in the form $(x + A)^2 + B$ where A and B are values to be stated.
    $(x + 5)^2 - 5$

Skills check 2

1. Write $x^2 - 4x + 5$ in the form $(x + A)^2 + B$ where A and B are values to be stated.
   $(x - 2)^2 + 1$

2. Solve the inequality $(x + 2)^2 \leq 5$. $-2 - \sqrt{5} \leq x \leq -2 + \sqrt{5}$

3. Given $f(x) = x^3 + 12x^2 + 43x + 32$, find the value of $f(-1)$ and write down a factor of $f(x)$.
   $f(-1) = 0 \; therefore \; (x + 1) is \; a \; factor$

4. Sketch the graph of $y = (x - 2)^2 - 5$, showing clearly the coordinates of the vertex. Vertex at (2,-5). Crosses y-axis at (0,-1)

5. A (3, -4) and B (-5, 2) are the ends of a diameter of a circle. Write down the coordinates of the centre of the circle. (-1,-1)

6. Given line A has equation $y = 3x + 7$, write down the equation of another line which is perpendicular to line A. Any line with the gradient -1/3

7. Simplify $\dfrac{\sqrt{11} + 1}{\sqrt{11} - 1}$. $\frac{6+\sqrt{11}}{5}$

8. Sketch the graph of $y = (x - 1)(2 - x)(2x + 1)$, showing clearly any intersections with the coordinate axes. Negative cubic graph crossing x-axis at (1,0), (2,0)(-1/2,0)

9. Factorise $-9x^2 + 18x - 5$. −(3x-1)(3x-5)

10. Solve the simultaneous equations
    $$y = 3x$$
    $$xy + x = 2$$

 $$x(3x) + x = 2$$
    $$3x^2 + x - 2 = 0$$
    $$(x + 1)(3x - 2) = 0$$
    $$x = -1, y = -3$$
    $$x = \frac{2}{3}, y = 3$$

3

Skills check 3

1. Solve $(2x-3)^2 = 2x$, expressing your answers in surd form. $x = \frac{7\pm\sqrt{13}}{4}$

2. Solve the inequality $2x^2 \langle 18$. $-3 < x < 3$

3. Find the remainder when $f(x) = x^3 - 4x^2 - 5x + 7$ is divided by $(x+2)$.
   $f(-2) = -8 - 16 + 10 + 7 = -7$

4. Sketch the graph of $(x-1)^2 + (y+2)^2 = 25$.
   Circle radius 5, centre (-1,2)

5. Simplify $\left(2p^2qr\right)^2 \times 4pq^2r$. $16p^5q^4r^3$

6. Write down the reciprocal of –5. -1/5

7. Find the mid-point of the line joining (3, -5) and (5, 3). (4,-1)

8. Find the gradient of the line joining (6, -5) and (9,3). Gradient =2

9. Factorise $12x^2 + 7x - 10$. (3x-2)(4x+5)

10. The diagram shows a square piece of card, with sides of length 6 cm. A smaller square of side x cm is cut from each corner as shown. The card is folded along the dotted lines to make an open box. Find an expression for the volume of the box in terms of x.

6 cm

6 cm

x

$x(6-2x)^2 = 4x^3 - 24x^2 + 36x$

4
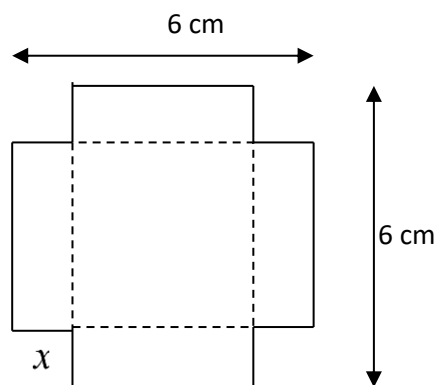
II. Short Exam Questions

Section 1- Equation of lines

1. For each of the following equations give the **gradient** and the **y-intercept** of each line.
   a) y=2x-7   2 and (0, -7)
   b) y+3x=8   3 and (0,8)
   c) 4y-2x+7=0   0.5 and (0,-7/4)

2. Find the equation of the line passing through the point (7, 10), and parallel to the line $8x + 4y = 3$. 8x + 4y = 96

3. Find the area of the triangle bounded by the coordinate axes and the line $3y - x + 12 = 0$.
   Area = 24 units^2

4. Find the points at which the following lines cut the coordinate axes:
   a. x-3y+2 = 0   (0,⅔) and (-2,0)
   b. 5x+9y-12 = 0 $(0, \frac{4}{3})$ and $(\frac{12}{5}, 0)$

5. Find the equations of these straight lines:
   a. Gradient = 3, and passing through (2,5).   y = 3x-1
   b. Gradient = ⅔, and passing through (5,-1)   2x-3y = 13
   c. Gradient = -4, and passing through (-2,13)   y+4x = 5

6. Find the equations of the line AB in each of the following cases:
   a. A(4,7) B(12,23)   y = 2x-1
   b. A(5,-1) B(3,5)   y= 14-3x
   c. A(3,10) B(3,27)   x = 3 (vertical line)
   d. A(-2,4) B(5,3)   x+7y-26 = 0   Variations of all of these are fine

7. Find the equation of the line passing through (6,-1) and parallel to the line 2x+3y = 4.
   2x+3y = 9

Section 2-
Section 2- Midpoints, distances and bisectors

1. Find the mid-point of the lines joining these pairs of points:
   a) (3,8) to (11,2)   (7,5)
   b) (-5,13) to (8,-13) (1.5,0)
   c) (-2,-7) to (0,-2)   (-1,-4.5)

2. For each of the following lines, find their gradient **m$_1$** and the gradient of their perpendicular **m$_2$**:
   a) $y = 7x + 5$ m$_1$ =7   m$_2$= -1/7
   b) $4y - x + 12 = 0$ m$_1$ = ¼, m$_2$ = -4
   c) $4x + 3y = 12$ m$_1$ = - 4/3, m$_2$ = 3/4

3. Find the equation of the line through (4,2) and perpendicular to the line 5y = 2x+15.
   2y+5x = 24

5

4. Find the equation of the perpendicular bisector of the line segment AB with A(-4,3) and B(4,7).
Equation of line y=2x+5
Midpoint at (0,5)
Equation of perpendicular bisector y= -1/2 x+c

5. Triangle ABC has vertices A(2,1), B(8,4), C(14, 1). Prove the triangle is isosceles. Need to show that two of the lines have the same length. AB and BC have length $\sqrt{45}$

6. Find the equation of the perpendicular bisector of the line segment AB with A(6,9) and B(10,1).
Equation of the line y= -2x+21
Midpoint (8,5)
Equation of the perpendicular y = 1/2x +1

7. The triangle PQR has corners P(1,1), Q(3,5) and R(5,-2).
Find the equation of the line perpendicular to PQ and passing through R.

Line PQ has equation y=2x-1

Perpendicular equation 2y+x=1

Section 3- Intersecting lines

1. Find the point of intersection of the following pairs of lines:
   a) $y = 3x - 1$ and $y = 5 - x$ (1.5,3.5)
   b) $y = x + 4$ and $3y - 2x + 8 = 0$ (-20,-16)
   c) $3x - 4y = 11$ and $5x + 2y = 14$ (3,-½)
   d) $3x + 4y = 10$ and $y = ½x - 5$ (6, -2)

2. The sides of the triangle ABC are given by the following lines:
   AB: x-2y = -1

   BC: 7x+6y = 53

   AC: 9x+2y = 11.

   a) Use simultaneous equations to find the coordinates of the three vertices of the triangle.
   A(1,1)   B(5,3)   C(-1,10)

   b) Hence show that the triangle is isosceles. Show that the length of two sides is the same. $\sqrt{85}$

3. The line *l* passes through the point (1,4) and is **perpendicular** to the line with the equation   2y = x - 7.
   a) Find an equation for *l*. 2x+y = 6

   Find the coordinates of the point where the lines meet. $\left(\frac{19}{5}, -\frac{8}{5}\right)$

4. Given that....

> The perpendicular bisector of the line joining any two points on the circumference of a circle will pass through the centre of the circle.

Given that the points A(-2,4), B(6,4) and C(5,-3) all

lie on the circumference of a circle:

a) Write down the perpendicular bisector of the line segment AB. $x = 2$

b) Work out the perpendicular bisector of the line segment AC. $y = x-1$

c) By solving simultaneously, find the centre of the circle. (2,1)

d) Calculate the radius of this circle. 5

A(-2,4)   B(6,4)

C(5,-3)
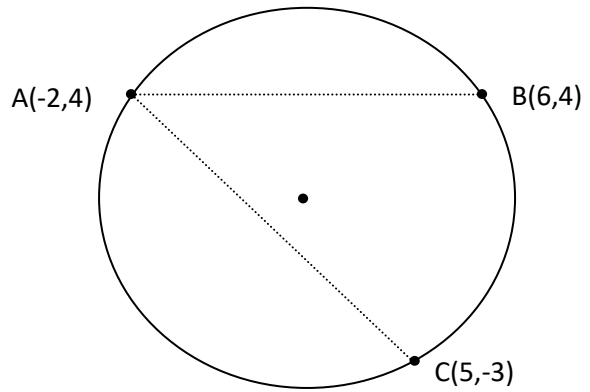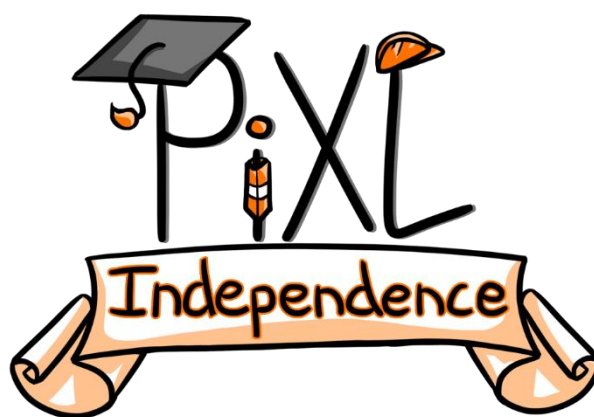
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