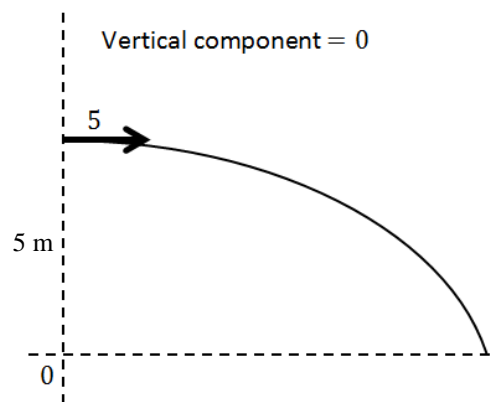


## Section 1: Introduction

### Solutions to Exercise level 1

1. (i) (a)



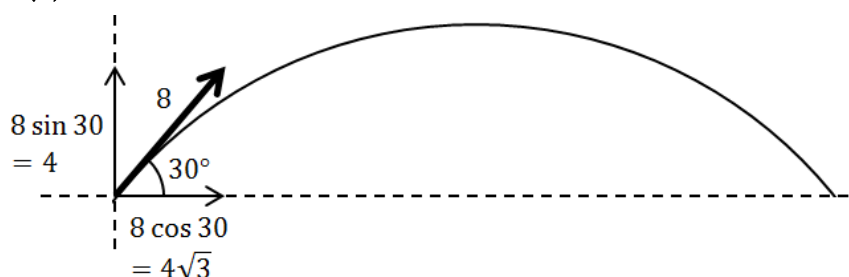
(b)  $\underline{v} = \underline{u} + \underline{a}t$

$$\underline{v} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t = \begin{pmatrix} 5 \\ -9.8t \end{pmatrix}$$

(c) From starting point  $\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2$   
using  $\underline{r}$  for the position of the projectile:

$$\begin{aligned} \underline{r} &= \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t^2 \\ &= \begin{pmatrix} 5t \\ 5 - 4.9t^2 \end{pmatrix} \end{aligned}$$

(ii) (a)



(b)  $\underline{v} = \underline{u} + \underline{a}t$

$$\underline{v} = \begin{pmatrix} 4\sqrt{3} \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t = \begin{pmatrix} 4\sqrt{3} \\ 4 - 9.8t \end{pmatrix}$$

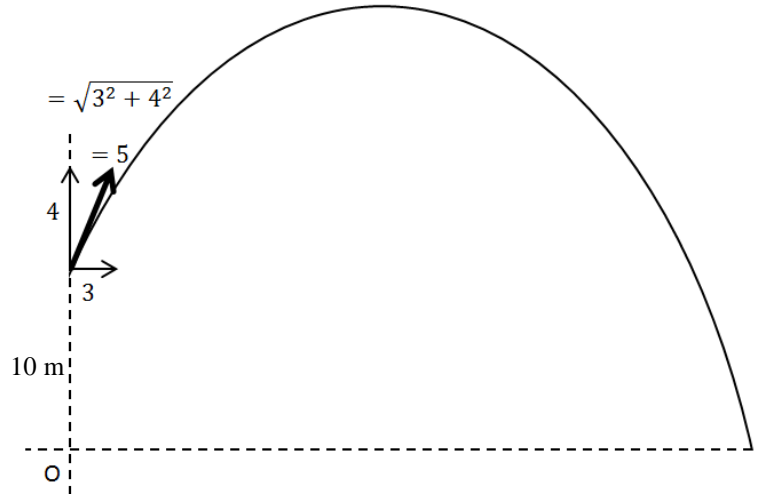
(c)  $\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2$

Since it starts at the origin,  $\underline{s}$  gives the position:

## AQA A level Maths Projectiles 1 Exercise solutions

$$\begin{aligned} \underline{s} &= \begin{pmatrix} 4\sqrt{3} \\ 4 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t^2 \\ &= \begin{pmatrix} 4\sqrt{3}t \\ 4t - 4.9t^2 \end{pmatrix} \end{aligned}$$

(iii) (a)



(b)  $\underline{v} = \underline{u} + \underline{a}t$

$$\underline{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t = \begin{pmatrix} 3 \\ 4 - 9.8t \end{pmatrix}$$

(c) From starting point  $\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2$   
using  $\underline{r}$  for the position of the projectile:

$$\begin{aligned} \underline{r} &= \begin{pmatrix} 0 \\ 10 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t^2 \\ &= \begin{pmatrix} 3t \\ 10 + 4t - 4.9t^2 \end{pmatrix} \end{aligned}$$

2. (i) (a) Looking at vertical motion:  $u = 5 \sin 60^\circ, v = 0, a = -9.8, t = ?$

$$v = u + at$$

$$0 = 5 \sin 60^\circ - 9.8t$$

$$t = \frac{5 \sin 60^\circ}{9.8} = 0.442 \text{ s (3 s.f.)}$$

(b)  $v^2 = u^2 + 2as$

$$0 = (5 \sin 60^\circ)^2 - 19.6s$$

$$s = \frac{(5 \sin 60^\circ)^2}{19.6} = 0.957 \text{ m (3 s.f.)}$$

$$\text{Maximum height} = 15 + s = 16.0 \text{ m (3 s.f.)}$$

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(ii) (a) Looking at vertical motion:  $u = 4, v = 0, a = -9.8, t = ?$

$$v = u + at$$

$$0 = 4 - 9.8t$$

$$t = \frac{4}{9.8} = 0.408 \text{ s (3 s.f.)}$$

(b)  $v^2 = u^2 + 2as$

$$0 = 4^2 - 19.6s$$

$$s = \frac{16}{19.6} = 0.816 \text{ m (3 s.f.)}$$

$$\text{Maximum height} = 3 + s = 3.82 \text{ m (3 s.f.)}$$

3. (i) Looking at vertical motion to calculate the time of flight:

$$s = 0, u = 4, a = -9.8, t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$0 = 4t - 4.9t^2$$

$$t(4.9t - 4) = 0$$

$$t = 0 \text{ initially or } t = \frac{4}{4.9} = 0.816 \text{ s (3 s.f.)}$$

Looking at horizontal motion:

Horizontal velocity is constant, so  $s = ut$

$$= 3 \times \frac{4}{4.9}$$

$$= 2.45 \text{ m (3 s.f.)}$$

(ii) Looking at vertical motion to calculate the time of flight:

$$s = 0, u = 1, a = -9.8, t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$0 = t - 4.9t^2$$

$$t(4.9t - 1) = 0$$

$$t = 0 \text{ initially or } t = \frac{1}{4.9} = 0.204 \text{ s (3 s.f.)}$$

Looking at horizontal motion:

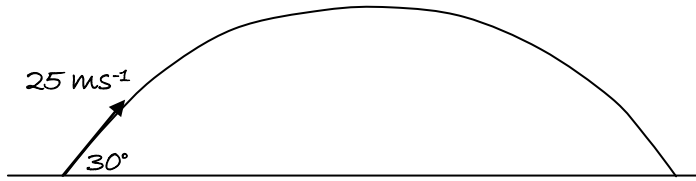
Horizontal velocity is constant, so  $s = ut$

$$= 5 \times \frac{1}{4.9}$$

$$= 1.02 \text{ m (3 s.f.)}$$

## AQA A level Maths Projectiles 1 Exercise solutions

4. (i)



$$\begin{aligned} \text{(ii)} \quad u_x &= 25 \cos 30^\circ = 12.5\sqrt{3} \\ u_y &= 25 \sin 30^\circ = 12.5 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad v_x &= 12.5\sqrt{3} && \text{(horizontal speed is constant)} \\ v_y &= u_y - gt \\ &= 12.5 - 9.8t \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad x &= u_x t \\ &= 12.5\sqrt{3}t \\ y &= u_y t - \frac{1}{2}gt^2 \\ &= 12.5t - 4.9t^2 \end{aligned}$$

5. (i) At maximum height  $v_y = 0$

$$\begin{aligned} \text{vertically: } v_y^2 &= u_y^2 - 2gh \\ 0 &= 12.5^2 - 2 \times 9.8h \\ h &= \frac{12.5^2}{19.6} = 7.97 \end{aligned}$$

Greatest height reached = 7.97 m.

(ii) The particle hits the ground when  $y = 0$ :

$$\begin{aligned} \text{vertically: } y &= 12.5t - 4.9t^2 \\ 0 &= t(12.5 - 4.9t) \\ t &= 0 \text{ or } t = \frac{12.5}{4.9} \end{aligned}$$

Time taken = 2.55 seconds.

(iii) Range = horizontal distance covered during time of flight

$$\begin{aligned} \text{Horizontally: } x &= 12.5\sqrt{3}t \\ &= 12.5\sqrt{3} \times \frac{12.5}{4.9} \\ &= 55.2 \end{aligned}$$

Horizontal range = 55.2 m.