

Name:

Class/Set:

Discrete - Simplex Method

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Please answer on separate paper.

1: Solve using the Simplex algorithm:

$$\begin{array}{r} \text{a) } \begin{array}{cccccc} P & x & y & s & t & \text{value} \\ \hline 0 & -5 & 5 & 1 & 0 & 20 \\ 0 & 9 & 0 & 0 & 1 & 45 \\ \hline 1 & -20 & -28 & 0 & 0 & 12 \end{array} \end{array}$$

$$\begin{array}{r} \text{b) } \begin{array}{cccccc} P & x & y & s & t & \text{value} \\ \hline 0 & -2 & 2 & 1 & 0 & 6 \\ 0 & 5 & 0 & 0 & 1 & 10 \\ \hline 1 & -6 & -6 & 0 & 0 & 16 \end{array} \end{array}$$

$$\begin{array}{r} \text{c) } \begin{array}{cccccc} P & x & y & s & t & \text{value} \\ \hline 0 & 0 & 1 & 1 & 0 & 8 \\ 0 & 8 & 0 & 0 & 1 & 8 \\ \hline 1 & -20 & -36 & 0 & 0 & 3 \end{array} \end{array}$$

$$\begin{array}{r} \text{d) } \begin{array}{cccccc} P & x & y & s & t & \text{value} \\ \hline 0 & 8 & 9 & 1 & 0 & 81 \\ 0 & 1 & -3 & 0 & 1 & 6 \\ \hline 1 & -4 & -35 & 0 & 0 & 36 \end{array} \end{array}$$

$$\begin{array}{r} \text{e) } \begin{array}{cccccc} P & x & y & s & t & \text{value} \\ \hline 0 & -1 & 6 & 1 & 0 & 42 \\ 0 & 8 & -4 & 0 & 1 & 16 \\ \hline 1 & -4 & -4 & 0 & 0 & 20 \end{array} \end{array}$$

$$\begin{array}{r} \text{f) } \begin{array}{cccccc} P & x & y & s & t & \text{value} \\ \hline 0 & 2 & 4 & 1 & 0 & 20 \\ 0 & 3 & -1 & 0 & 1 & 9 \\ \hline 1 & -10 & -35 & 0 & 0 & 2 \end{array} \end{array}$$

2: Solve using the Simplex algorithm:

$$\begin{array}{r} \text{a) } \begin{array}{ccccccc} P & x & y & z & s & t & u & \text{value} \\ \hline 0 & 8 & 4 & 14 & 1 & 0 & 0 & 3 \\ 0 & 6 & 28 & 6 & 0 & 1 & 0 & 14 \\ 0 & 12 & 10 & 9 & 0 & 0 & 1 & 20 \\ \hline 1 & -16 & -45 & -10 & 0 & 0 & 0 & 40 \end{array} \end{array}$$

$$\begin{array}{r} \text{b) } \begin{array}{ccccccc} P & x & y & z & s & t & u & \text{value} \\ \hline 0 & 16 & 24 & 2 & 1 & 0 & 0 & 20 \\ 0 & 4 & 50 & 20 & 0 & 1 & 0 & 21 \\ 0 & 15 & 8 & 18 & 0 & 0 & 1 & 16 \\ \hline 1 & -18 & -20 & -4 & 0 & 0 & 0 & 5 \end{array} \end{array}$$

$$\begin{array}{r} \text{c) } \begin{array}{ccccccc} P & x & y & z & s & t & u & \text{value} \\ \hline 0 & 25 & 9 & 8 & 1 & 0 & 0 & 3 \\ 0 & 4 & 10 & 3 & 0 & 1 & 0 & 30 \\ 0 & 21 & 25 & 20 & 0 & 0 & 1 & 10 \\ \hline 1 & -5 & -35 & -4 & 0 & 0 & 0 & 30 \end{array} \end{array}$$

$$\begin{array}{r} \text{d) } \begin{array}{ccccccc} P & x & y & z & s & t & u & \text{value} \\ \hline 0 & 7 & 12 & 14 & 1 & 0 & 0 & 14 \\ 0 & 6 & 9 & 4 & 0 & 1 & 0 & 12 \\ 0 & 8 & 35 & 8 & 0 & 0 & 1 & 1 \\ \hline 1 & -10 & -4 & -15 & 0 & 0 & 0 & 7 \end{array} \end{array}$$

$$\begin{array}{r} \text{e) } \begin{array}{ccccccc} P & x & y & z & s & t & u & \text{value} \\ \hline 0 & 8 & 6 & 2 & 1 & 0 & 0 & 6 \\ 0 & 9 & 28 & 4 & 0 & 1 & 0 & 14 \\ 0 & 12 & 2 & 15 & 0 & 0 & 1 & 6 \\ \hline 1 & -2 & -18 & -10 & 0 & 0 & 0 & 40 \end{array} \end{array}$$

$$\begin{array}{r} \text{f) } \begin{array}{ccccccc} P & x & y & z & s & t & u & \text{value} \\ \hline 0 & 7 & 18 & 21 & 1 & 0 & 0 & 12 \\ 0 & 10 & 6 & 12 & 0 & 1 & 0 & 30 \\ 0 & 20 & 2 & 12 & 0 & 0 & 1 & 7 \\ \hline 1 & -9 & -5 & -20 & 0 & 0 & 0 & 15 \end{array} \end{array}$$

Answers: Discrete - Simplex Method

1: a) Iteration 1:

P	x	y	s	t	value
0	-5	5	1	0	20
0	9	0	0	1	45
1	-20	-28	0	0	12

Iteration 2:

P	x	y	s	t	value
0	-1	1	$\frac{1}{5}$	0	4
0	9	0	0	1	45
1	-48	0	$5\frac{3}{5}$	0	124

Optimal solution:

P	x	y	s	t	value
0	0	1	$\frac{1}{5}$	$\frac{1}{9}$	9
0	1	0	0	$\frac{1}{9}$	5
1	0	0	$5\frac{3}{5}$	$5\frac{1}{3}$	364

$$P = 364, x = 5, y = 9$$

c) Iteration 1:

P	x	y	s	t	value
0	0	1	1	0	8
0	8	0	0	1	8
1	-20	-36	0	0	3

Iteration 2:

P	x	y	s	t	value
0	0	1	1	0	8
0	8	0	0	1	8
1	-20	0	36	0	291

Optimal solution:

P	x	y	s	t	value
0	0	1	1	0	8
0	1	0	0	$\frac{1}{8}$	1
1	0	0	36	$2\frac{1}{2}$	311

$$P = 311, x = 1, y = 8$$

b) Iteration 1:

P	x	y	s	t	value
0	-2	2	1	0	6
0	5	0	0	1	10
1	-6	-6	0	0	16

Iteration 2:

P	x	y	s	t	value
0	0	2	1	$\frac{2}{5}$	10
0	1	0	0	$\frac{1}{5}$	2
1	0	-6	0	$1\frac{1}{5}$	28

Optimal solution:

P	x	y	s	t	value
0	0	1	$\frac{1}{2}$	$\frac{1}{5}$	5
0	1	0	0	$\frac{1}{5}$	2
1	0	0	3	$2\frac{2}{5}$	58

$$P = 58, x = 2, y = 5$$

d) Iteration 1:

P	x	y	s	t	value
0	8	9	1	0	81
0	1	-3	0	1	6
1	-4	-35	0	0	36

Optimal solution:

P	x	y	s	t	value
0	$\frac{8}{9}$	1	$\frac{1}{9}$	0	9
0	$3\frac{2}{3}$	0	$\frac{1}{3}$	1	33
1	$27\frac{1}{9}$	0	$3\frac{8}{9}$	0	351

$$P = 351, x = 0, y = 9$$

e) Iteration 1:

P	x	y	s	t	value
0	-1	6	1	0	42
0	8	-4	0	1	16
1	-4	-4	0	0	20

Iteration 2:

P	x	y	s	t	value
0	0	$5\frac{1}{2}$	1	$\frac{1}{8}$	44
0	1	$-\frac{1}{2}$	0	$\frac{1}{8}$	2
1	0	-6	0	$\frac{1}{2}$	28

Optimal solution:

P	x	y	s	t	value
0	0	1	$\frac{2}{11}$	$\frac{1}{44}$	8
0	1	0	$\frac{1}{11}$	$\frac{3}{22}$	6
1	0	0	$1\frac{1}{11}$	$\frac{7}{11}$	76

$P = 76, x = 6, y = 8$

f) Iteration 1:

P	x	y	s	t	value
0	2	4	1	0	20
0	3	-1	0	1	9
1	-10	-35	0	0	2

Optimal solution:

P	x	y	s	t	value
0	$\frac{1}{2}$	1	$\frac{1}{4}$	0	5
0	$3\frac{1}{2}$	0	$\frac{1}{4}$	1	14
1	$7\frac{1}{2}$	0	$8\frac{3}{4}$	0	177

$P = 177, x = 0, y = 5$

2: a) Iteration 1:

P	x	y	z	s	t	u	value
0	8	4	14	1	0	0	3
0	6	28	6	0	1	0	14
0	12	10	9	0	0	1	20
1	-16	-45	-10	0	0	0	40

Iteration 2:

P	x	y	z	s	t	u	value
0	$7\frac{1}{7}$	0	$13\frac{1}{7}$	1	$-\frac{1}{7}$	0	1
0	$\frac{3}{14}$	1	$\frac{3}{14}$	0	$\frac{1}{28}$	0	$\frac{1}{2}$
0	$9\frac{6}{7}$	0	$6\frac{6}{7}$	0	$-\frac{5}{14}$	1	15
1	$-6\frac{5}{14}$	0	$-\frac{5}{14}$	0	$1\frac{17}{28}$	0	$62\frac{1}{2}$

Optimal solution:

P	x	y	z	s	t	u	value
0	1	0	$1\frac{21}{25}$	$\frac{7}{50}$	$-\frac{1}{50}$	0	$\frac{7}{50}$
0	0	1	$-\frac{9}{50}$	$-\frac{3}{100}$	$\frac{1}{25}$	0	$\frac{47}{100}$
0	0	0	$-11\frac{7}{25}$	$-\frac{19}{50}$	$-\frac{4}{25}$	1	$13\frac{31}{50}$
1	0	0	$11\frac{17}{50}$	$\frac{89}{100}$	$1\frac{12}{25}$	0	$63\frac{39}{100}$

$$P = 63\frac{39}{100}, x = \frac{7}{50}, y = \frac{47}{100}$$

b) Iteration 1:

P	x	y	z	s	t	u	value
0	16	24	2	1	0	0	20
0	4	50	20	0	1	0	21
0	15	8	18	0	0	1	16
1	-18	-20	-4	0	0	0	5

Iteration 2:

P	x	y	z	s	t	u	value
0	$14\frac{2}{25}$	0	$-7\frac{3}{5}$	1	$-\frac{12}{25}$	0	$9\frac{23}{25}$
0	$\frac{2}{25}$	1	$\frac{2}{5}$	0	$\frac{1}{50}$	0	$\frac{21}{50}$
0	$14\frac{9}{25}$	0	$14\frac{4}{5}$	0	$-\frac{4}{25}$	1	$12\frac{16}{25}$
1	$-16\frac{2}{5}$	0	4	0	$\frac{2}{5}$	0	$13\frac{2}{5}$

Iteration 3:

P	x	y	z	s	t	u	value
0	1	0	$-\frac{95}{176}$	$\frac{25}{352}$	$-\frac{3}{88}$	0	$\frac{31}{44}$
0	0	1	$\frac{39}{88}$	$-\frac{1}{176}$	$\frac{1}{44}$	0	$\frac{4}{11}$
0	0	0	$22\frac{97}{176}$	$-\frac{7}{352}$	$\frac{29}{88}$	1	$2\frac{23}{44}$
1	0	0	$-4\frac{75}{88}$	$\frac{29}{176}$	$-\frac{7}{44}$	0	$24\frac{21}{22}$

Iteration 4:

P	x	y	z	s	t	u	value
0	1	0	0	$\frac{185}{3969}$	$-\frac{104}{3969}$	$\frac{95}{3969}$	$\frac{1012}{1323}$
0	0	1	0	$\frac{19}{1323}$	$\frac{43}{2646}$	$-\frac{26}{1323}$	$\frac{277}{882}$
0	0	0	1	$-\frac{359}{7938}$	$\frac{58}{3969}$	$\frac{176}{3969}$	$\frac{148}{1323}$
1	0	0	0	$\frac{536}{567}$	$-\frac{50}{567}$	$\frac{122}{567}$	$25\frac{94}{189}$

Optimal solution:

P	x	y	z	s	t	u	value
0	1	0	$1\frac{23}{29}$	$-\frac{1}{29}$	0	$\frac{3}{29}$	$\frac{28}{29}$
0	0	1	$-1\frac{13}{116}$	$\frac{15}{232}$	0	$-\frac{2}{29}$	$\frac{11}{58}$
0	0	0	$68\frac{25}{58}$	$-3\frac{11}{116}$	1	$3\frac{1}{29}$	$7\frac{19}{29}$
1	0	0	$6\frac{1}{29}$	$\frac{39}{58}$	0	$\frac{14}{29}$	$26\frac{5}{29}$

$$P = 26\frac{5}{29}, x = \frac{28}{29}, y = \frac{11}{58}$$

c) Iteration 1:

P	x	y	z	s	t	u	value
0	25	9	8	1	0	0	3
0	4	10	3	0	1	0	30
0	21	25	20	0	0	1	10
1	-5	-35	-4	0	0	0	30

Optimal solution:

P	x	y	z	s	t	u	value
0	$2\frac{7}{9}$	1	$\frac{8}{9}$	$\frac{1}{9}$	0	0	$\frac{1}{3}$
0	$-23\frac{7}{9}$	0	$-5\frac{8}{9}$	$-1\frac{1}{9}$	1	0	$26\frac{2}{3}$
0	$-48\frac{4}{9}$	0	$-2\frac{2}{9}$	$-2\frac{7}{9}$	0	1	$1\frac{2}{3}$
1	$92\frac{2}{9}$	0	$27\frac{1}{9}$	$3\frac{8}{9}$	0	0	$41\frac{2}{3}$

$$P = 41\frac{2}{3}, x = 0, y = \frac{1}{3}$$

d) Iteration 1:

P	x	y	z	s	t	u	value
0	7	12	14	1	0	0	14
0	6	9	4	0	1	0	12
0	8	35	8	0	0	1	1
1	-10	-4	-15	0	0	0	7

Optimal solution:

P	x	y	z	s	t	u	value
0	-7	$-49\frac{1}{4}$	0	1	0	$-1\frac{3}{4}$	$12\frac{1}{4}$
0	2	$-8\frac{1}{2}$	0	0	1	$-\frac{1}{2}$	$11\frac{1}{2}$
0	1	$4\frac{3}{8}$	1	0	0	$\frac{1}{8}$	$\frac{1}{8}$
1	5	$61\frac{5}{8}$	0	0	0	$1\frac{7}{8}$	$8\frac{7}{8}$

$$P = 8\frac{7}{8}, x = 0, y = 0$$

e) Iteration 1:

P	x	y	z	s	t	u	value
0	8	6	2	1	0	0	6
0	9	28	4	0	1	0	14
0	12	2	15	0	0	1	6
1	-2	-18	-10	0	0	0	40

Iteration 2:

P	x	y	z	s	t	u	value
0	$6\frac{1}{14}$	0	$1\frac{1}{7}$	1	$-\frac{3}{14}$	0	3
0	$\frac{9}{28}$	1	$\frac{1}{7}$	0	$\frac{1}{28}$	0	$\frac{1}{2}$
0	$11\frac{5}{14}$	0	$14\frac{5}{7}$	0	$-\frac{1}{14}$	1	5
1	$3\frac{11}{14}$	0	$-7\frac{3}{7}$	0	$\frac{9}{14}$	0	49

Optimal solution:

P	x	y	z	s	t	u	value
0	$5\frac{39}{206}$	0	0	1	$-\frac{43}{206}$	$-\frac{8}{103}$	$2\frac{63}{103}$
0	$\frac{87}{412}$	1	0	0	$\frac{15}{412}$	$-\frac{1}{103}$	$\frac{93}{206}$
0	$\frac{159}{206}$	0	1	0	$-\frac{1}{206}$	$\frac{7}{103}$	$\frac{35}{103}$
1	$9\frac{107}{206}$	0	0	0	$\frac{125}{206}$	$\frac{52}{103}$	$51\frac{54}{103}$

$$P = 51\frac{54}{103}, x = 0, y = \frac{93}{206}$$

f) Iteration 1:

P	x	y	z	s	t	u	value
0	7	18	21	1	0	0	12
0	10	6	12	0	1	0	30
0	20	2	12	0	0	1	7
1	-9	-5	-20	0	0	0	15

Iteration 2:

P	x	y	z	s	t	u	value
0	$\frac{1}{3}$	$\frac{6}{7}$	1	$\frac{1}{21}$	0	0	$\frac{4}{7}$
0	6	$-4\frac{2}{7}$	0	$-\frac{4}{7}$	1	0	$23\frac{1}{7}$
0	16	$-8\frac{2}{7}$	0	$-\frac{4}{7}$	0	1	$\frac{1}{7}$
1	$-2\frac{1}{3}$	$12\frac{1}{7}$	0	$\frac{20}{21}$	0	0	$26\frac{3}{7}$

Optimal solution:

P	x	y	z	s	t	u	value
0	0	$1\frac{5}{168}$	1	$\frac{5}{84}$	0	$-\frac{1}{48}$	$\frac{191}{336}$
0	0	$-1\frac{5}{28}$	0	$-\frac{5}{14}$	1	$-\frac{3}{8}$	$23\frac{5}{56}$
0	1	$-\frac{29}{56}$	0	$-\frac{1}{28}$	0	$\frac{1}{16}$	$\frac{1}{112}$
1	0	$10\frac{157}{168}$	0	$\frac{73}{84}$	0	$\frac{7}{48}$	$26\frac{151}{336}$

$$P = 26\frac{151}{336}, x = \frac{1}{112}, y = 0$$