Those questions in this exercise where the context is drawn from real life involve working with mathematical models.

① The forces $\mathbf{F}_1 = 4\mathbf{i} - 5\mathbf{j}$ and $\mathbf{F}_2 = 2\mathbf{i} + \mathbf{j}$, in newtons, act on a particle of mass 4 kg.

(i) Find the acceleration of the particle in component form.

(ii) Find the magnitude of the particle's acceleration.

② Two forces $\mathbf{P}_1$ and $\mathbf{P}_2$ act on a particle of mass 2 kg giving it an acceleration of $5\mathbf{i} + 5\mathbf{j}$ (in $\mathrm{m\,s^{-2}}$).

(i) If $\mathbf{P}_1 = 6\mathbf{i} - \mathbf{j}$ (in newtons), find $\mathbf{P}_2$.

(ii) If instead $\mathbf{P}_1$ and $\mathbf{P}_2$ both act in the same direction but $\mathbf{P}_1$ is four times as big as $\mathbf{P}_2$ find both forces.

③ Figure 19.65 shows a girl pulling a sledge at steady speed across level snow-covered ground using a rope which makes an angle of 30° to the horizontal. The mass of the sledge is 8 kg and there is a resistance force of 10 N.

Figure 19.65

(i) Draw a diagram showing the forces acting on the sledge.

(ii) Find the magnitude of the tension in the rope.

The girl comes to an area of ice where the resistance force on the sledge is only 2 N. She continues to pull the sledge with the same force as before and with the rope still taut at 30°.

(iii) What acceleration must the girl have in order to do this?

(iv) How long will it take to double her initial speed of $0.4\,\mathrm{m\,s^{-1}}$?

④ A block of mass 12 kg is placed on a smooth plane inclined at 40° to the horizontal. It is connected by a light inextensible string, which passes over a smooth pulley at the top of the plane, to a mass of 7 kg hanging freely (Figure 19.66). Find the common acceleration and the tension in the string.

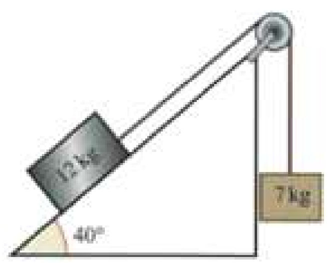


Figure 19.66

⑤ Figure 19.67 shows a situation which has arisen between two anglers, Davies and Jones, standing at the ends of adjacent jetties. Their lines have become entangled under the water with the result that they have both hooked the same fish, which has a mass 1.9 kg. Both are reeling in their lines as hard as they can in order to claim the fish.

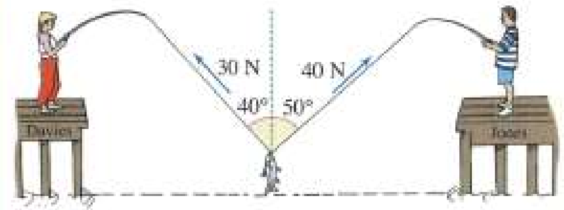


Figure 19.67

(i) Draw a diagram showing the forces acting on the fish.

(ii) Resolve the tensions in both anglers' lines into horizontal and vertical components and so find the total force acting on the fish.

(iii) Find the magnitude and direction of the acceleration of the fish.

(iv) At this point Davies' line breaks. What happens to the fish?

⑥ A crate of mass 30 kg is being pulled up a smooth slope inclined at 30° to the horizontal by a rope which is parallel to the slope. The crate has acceleration $0.75\,\mathrm{m\,s^{-2}}$.

(i) Draw a diagram showing the forces acting on the crate and the direction of its acceleration.

(ii) Resolve the forces in directions parallel and perpendicular to the slope.

(iii) Find the tension in the rope.

(iv) The rope suddenly snaps. What happens to the crate?

⑦ Two toy trucks are travelling down a slope inclined at an angle of 5° to the horizontal (Figure 19.68). Truck A has a mass of 6 kg, truck B has a mass of 4 kg.

Figure 19.68

The trucks are linked by a light rigid rod which is parallel to the slope.

The resistances to motion of the trucks are 2.5 N for truck A and 1.5 N for truck B.

The initial speed of the trucks is $2\,\mathrm{m\,s^{-1}}$.

Calculate the speed of the trucks after 3 seconds and also the force in the rod connecting the trucks, stating whether the rod is in tension or in thrust.

⑧ A car of mass 1000 kg is towing a trailer of mass 250 kg along a slope of 1 in 20 (i.e. at an angle θ to the horizontal and $\sin\theta = \frac{1}{20}$). The driving force of the engine is 2500 N and there are resistances to the motion of both the car, 700 N, and the trailer, 300 N.

Find the acceleration and the tension in the tow bar.

⑨ A cyclist of mass 60 kg rides a cycle of mass 7 kg. The greatest forward force that she can produce is 200 N but she is subject to air resistance and friction totalling 50 N.

(i) Draw a diagram showing the forces acting on the cyclist when she is going uphill.

(ii) What is the angle of the steepest slope that she can ascend?

The cyclist reaches a slope of 8° with a speed of $5\,\mathrm{m\,s^{-1}}$ and rides as hard as she can up it.

(iii) Find her acceleration and the distance she travels in 5 s.

(iv) What is her speed now?

⑩ A builder is demolishing the chimney of a house and slides the old bricks down to the ground on a straight chute 10 m long inclined at 42° to the horizontal. Each brick has a mass of 3 kg.

(i) Draw a diagram showing the forces acting on a brick as it slides down the chute, assuming the chute to have a flat cross section and a smooth surface.

(ii) Find the acceleration of the brick.

(iii) Find the time the brick takes to reach the ground.

In fact the chute is not smooth and the brick takes 3 s to reach the ground.

(iv) Find the frictional force acting on the brick, assuming it to be constant.

⑪ A box of mass 80 kg is to be pulled along a horizontal floor by means of a light rope. The rope is pulled with a force of 100 N and the rope is inclined at 20° to the horizontal, as shown in Figure 19.69.

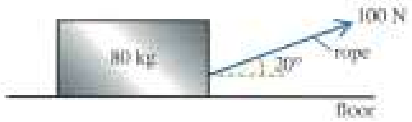


Figure 19.69

(i) Explain briefly why the box cannot be in equilibrium if the floor is smooth.

In fact the floor is not smooth and the box is in equilibrium.

(ii) Draw a diagram showing all the external forces acting on the box.

(iii) Calculate the frictional force between the box and the floor and also the normal reaction of the floor on the box, giving your answers correct to three significant figures.

The maximum value of the frictional force between the box and the floor is 120 N and the box is now pulled along the floor with the rope always inclined at 20° to the horizontal.

(iv) Calculate the force with which the rope must be pulled for the box to move at a constant speed. Give your answers correct to three significant figures.

(v) Calculate the acceleration of the box if the rope is pulled with a force of 140 N.

[MEI]

⑫ A block of mass 5 kg is at rest on a plane which is inclined at 30° to the horizontal. A light inelastic string is attached to the block, passes over a smooth pulley and supports a mass m kg which hangs freely. The part of the string between the block and the pulley is parallel to a line of greatest slope of the plane. A friction force of 15 N opposes the motion of the block. Figure 19.70 shows the block when it is slipping up the plane at a constant speed.

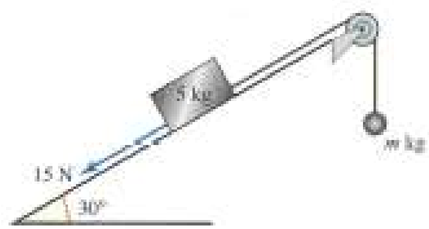


Figure 19.70

Give your answers to two significant figures.

(i) Copy the diagram and mark in all the forces acting on the block and the hanging mass, including the tension in the string.

(ii) Calculate the value of m when the block slides up the plane at a constant speed and find the tension in the string.

(iii) Calculate the acceleration of the system when $m = 6$ kg and find the tension in the string in this case. [MEI]

⑬ A sledge is found to travel with uniform speed down a slope of 1 in 50 (i.e. at an angle θ with the horizontal such that $\sin\theta = \frac{1}{50}$). If the sledge starts from the bottom of the same slope with a speed of $2\,\text{m s}^{-1}$, how far will it travel up the slope before coming to rest?

⑭ A train of mass 200 tonnes is travelling uniformly on level ground at $10\,\text{m s}^{-1}$ when it begins an ascent of 1 in 50. The driving force exerted by the engine is equal to 25 kN and the resistance force on the train is a constant 10 kN. Show that the train comes to a standstill after climbing for 413 m.

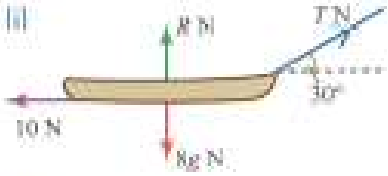
⑮ A railway truck runs down a slope of 1 in 250 and at the foot continues along level ground. Find how far it will run on the level if the speed was a constant $5\,\text{m s}^{-1}$ on the slope and the resistance is unchanged on the level.

Solutions

1 (i) $1.5\mathbf{i} - \mathbf{j}$
(ii) $1.80\,\text{m s}^{-2}$

2 (i) $4\mathbf{i} + 11\mathbf{j}$
(ii) $8\mathbf{i} + 8\mathbf{j}$, $2\mathbf{i} + 2\mathbf{j}$

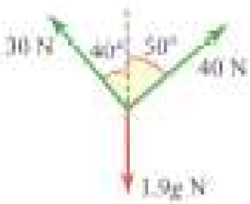
3 (i)



(ii) 11.55 N
(iii) $1\,\text{m s}^{-2}$
(iv) 0.4 s

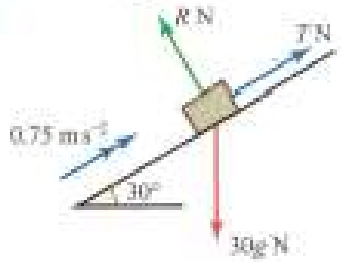
4 $0.37\,\text{m s}^{-2}$ down the plane, 71.2 N

5 (i)



(ii) 11.4 N, 30.1 N
(iii) $16.9\,\text{m s}^{-2}$ at 69°
(iv) The fish swings sideways as it moves up towards Jones.

6 (i)

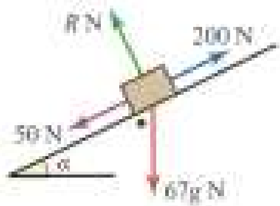


(ii) $T\mathbf{i}$, $R\mathbf{j}$,
$-30g\sin 30°\mathbf{i} - 30g\cos 30°\mathbf{j}$
(iii) 169.5 N
(iv) The crate slows down to a stop and then starts sliding down the slope.

7 $3.36\,\text{m s}^{-1}$, 0.1 N in compression

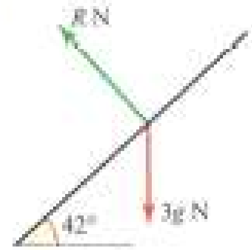
8 $0.71\,\text{m s}^{-2}$, 600 N

9 (i)



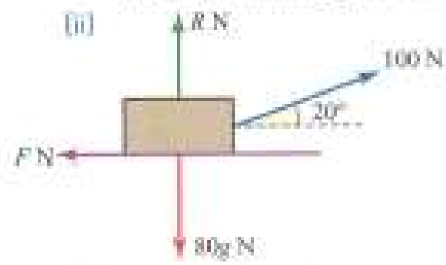
(ii) 13.2°
(iii) $0.87\,\text{m s}^{-2}$, 35.9 m
(iv) $9.37\,\text{m s}^{-1}$

10 (i)



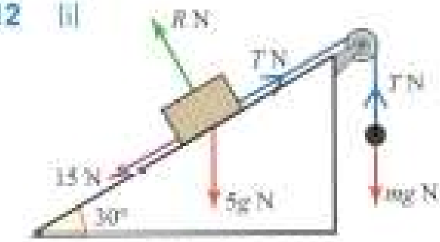
(ii) $6.56\,\text{m s}^{-2}$
(iii) 1.75 s
(iv) 13.0 N

11 (i) Horizontal component of tension in the rope needs a balancing force.
(ii)



(iii) 94.0 N, 750 N
(iv) 128 N
(v) $0.144\,\text{m s}^{-2}$

12 (i)



(ii) 4.03 kg, 39.5 N
(iii) $1.75\,\text{m s}^{-2}$, 48.3 N

13 5.10 m

14 $a = -0.121\,\text{m s}^{-2}$
$\Rightarrow s = 413\,\text{m}$

15 319 m