Vectors – Assessment Practice

1   Vector $\mathbf{p} = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$ and vector $\mathbf{q} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

   a   Evaluate $3\mathbf{p} + 5\mathbf{q}$. Select the correct answer.

     A $\begin{pmatrix} 33 \\ 7 \end{pmatrix}$ **B** $\begin{pmatrix} 21 \\ 7 \end{pmatrix}$ **C** $\begin{pmatrix} 3 \\ 17 \end{pmatrix}$ **D** $\begin{pmatrix} 3 \\ 23 \end{pmatrix}$ **[1 mark]**

   b   Calculate the unit vector $\hat{\mathbf{q}}$, in the direction of $\mathbf{q}$. Select the correct answer.

     A $\begin{pmatrix} -0.6 \\ 0.8 \end{pmatrix}$ **B** $\begin{pmatrix} -15 \\ 20 \end{pmatrix}$ **C** $\begin{pmatrix} -1.5 \\ 1.5 \end{pmatrix}$ **D** $\begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix}$ **[1]**

2   Four vectors, **a**, **b**, **c** and **d** have a resultant of 0. $\mathbf{a} = 2\mathbf{i} + 7\mathbf{j}$, $\mathbf{b} = 3\mathbf{i} - 10\mathbf{j}$ and $\mathbf{c} = 5\mathbf{i} - 21\mathbf{j}$

   a   Evaluate **d**. Select the correct answer.

     A $\begin{pmatrix} 10 \\ 24 \end{pmatrix}$ **B** $-10\mathbf{i} + 24\mathbf{j}$ **C** $10\mathbf{i} - 24\mathbf{j}$ **D** $\begin{pmatrix} 24 \\ 10 \end{pmatrix}$ **[1]**

   b   Evaluate $|\mathbf{d}|$. Select the correct answer.

     A   676   **B** $\pm\sqrt{676}$ **C**   26   **D** $\sqrt{476}$ **[1]**

3   A sailing boat starts from buoy A and sails 800 metres on a bearing of 032° to buoy B. It then sails 1200 metres on a bearing of 294° to buoy C. Work out

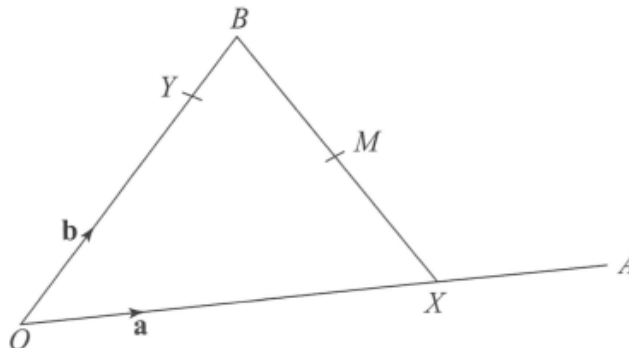   **a**   The distance AC **[6]**

   b   The bearing of C from A **[4]**

4   Given vectors $\mathbf{p} = x\mathbf{i} + 2\mathbf{j}$, $\mathbf{q} = 3\mathbf{i} + y\mathbf{j}$ and $\mathbf{r} = 4\mathbf{i} + 6\mathbf{j}$, evaluate

   a x and y if $\mathbf{r} = \mathbf{p} - \mathbf{q}$ **[3]**

   b x if **p** and **r** are parallel.   **[2]**

5   The diagram shows two vectors, **a** and **b**, where $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point X lies on OA where $OX:XA = 2:1$. The point Y lies on OB where $OY:YB = 3:1$. M is the midpoint of XB

   a   Express these vectors in terms of **a** and **b**   **[11]**

     i $\overrightarrow{OX}$ **ii** $\overrightarrow{OY}$ **iii** $\overrightarrow{OM}$ **iv** $\overrightarrow{AM}$ **v** $\overrightarrow{MY}$

   b   Use your answer to **a** to prove that A, M and Y are collinear, and work out the ratio $AM:MY$ **[2]**

6 Vectors $\mathbf{a} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$ and vector $\mathbf{b} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

Work out the magnitude and direction of the resultant of $\mathbf{a}$ and $\mathbf{b}$ [6]

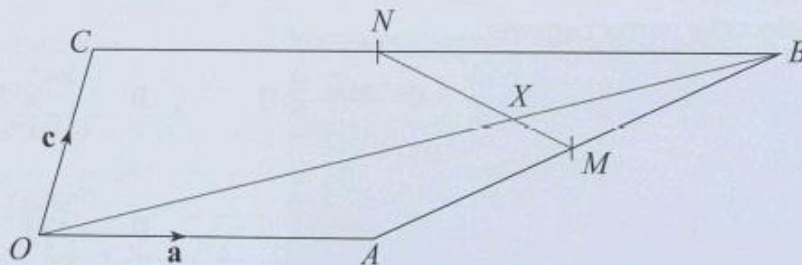**7** A is the point $(-2, 5)$, B is the point $(1, 3)$ and C is the point $(10, -3)$

   a Write down   **i** $\overrightarrow{AB}$ **ii** $\overrightarrow{BC}$ [2]

   b Prove that A, B and C are collinear. [2]

8 Two vectors, $\mathbf{a}$ and $\mathbf{b}$, are given by $\mathbf{a} = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Given $|\mathbf{a} + \lambda\mathbf{b}| = 13$, work out the possible values of the scalar λ [6]

9 The diagram shows a trapezium $OABC$ where $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = \mathbf{c}$, and $\overrightarrow{CB} = 2\mathbf{a}$
M is the midpoint of AB, and N is the midpoint of BC

   a Express these vectors in terms of $\mathbf{a}$ and $\mathbf{c}$ [7]

     i $\overrightarrow{OB}$ **ii** $\overrightarrow{ON}$ **iii** $\overrightarrow{AB}$ **iv** $\overrightarrow{OM}$ **v** $\overrightarrow{MN}$

The line OB meets the line MN at the point X

   b Evaluate $\overrightarrow{OX}$ [8]

10 Points P, Q, R and S have position vectors $\mathbf{p} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$, $\mathbf{r} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ and $\mathbf{s} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$ [7]

Prove that the quadrilateral $PQRS$ is a parallelogram.

11 Vectors $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ are given by $\mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\mathbf{y} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $\mathbf{z} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

   a Prove that $\mathbf{x} + 3\mathbf{y}$ is parallel to $\mathbf{z}$ [3]

   b Work out the value of the integer c, for which $\mathbf{x} + c\mathbf{z}$ is parallel to $\mathbf{y}$ [4]

12 Two forces act on an object: a 2 N force on a bearing of 030° and a 3.5 N force.
The resultant of the two forces acts in a northerly direction. Work out

   a the direction of the 3.5 N force, [6]

   b the magnitude of the resultant force. [2]

Assessment 6 Solutions

1 a C   b A
2 a B   b C
3 a 1346 m   b 330°
4 a $x = 7, y = -4$ b $x = \dfrac{4}{3}$

5 a i $\dfrac{2}{3}\mathbf{a}$ ii $\dfrac{3}{4}\mathbf{b}$ iii $\dfrac{1}{3}\mathbf{a} + \dfrac{1}{2}\mathbf{b}$

   iv $-\dfrac{2}{3}\mathbf{a} + \dfrac{1}{2}\mathbf{b}$ v $-\dfrac{1}{3}\mathbf{a} + \dfrac{1}{4}\mathbf{b}$

   b 2:1

6 $3\sqrt{5}, 116.6°$

7 a i $3\mathbf{i} - 2\mathbf{j}$ ii $9\mathbf{i} - 6\mathbf{j}$

   b $\overrightarrow{BC} = 3(3\mathbf{i} - 2\mathbf{j})$

   Since $\overrightarrow{BC}$ is a multiple of $\overrightarrow{AB}$, $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel, and since they have a point in common, A, B and C are collinear.

8 $\lambda = -4.8$ or 4

9 a i $2\mathbf{a} + \mathbf{c}$ ii $\mathbf{a} + \mathbf{c}$ iii $\mathbf{a} + \mathbf{c}$

   iv $\dfrac{3}{2}\mathbf{a} + \dfrac{1}{2}\mathbf{c}$ v $-\dfrac{1}{2}\mathbf{a} + \dfrac{1}{2}\mathbf{c}$

   b $\dfrac{4}{3}\mathbf{a} + \dfrac{2}{3}\mathbf{c}$

10 $\overrightarrow{PQ} = \mathbf{q} - \mathbf{p} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 6 \\ 3 \end{pmatrix} = \begin{pmatrix} -9 \\ -8 \end{pmatrix}$

   $\overrightarrow{SR} = \begin{pmatrix} -9 \\ -8 \end{pmatrix}$

   $\overrightarrow{QR} = \mathbf{r} - \mathbf{q} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

   $\overrightarrow{PS} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

   Since $\overrightarrow{PQ} = \overrightarrow{SR}$ and $\overrightarrow{QR} = \overrightarrow{PS}$, $PQRS$ is a parallelogram

11 a $\mathbf{x} + 3\mathbf{y} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 15 \\ 9 \end{pmatrix} = \begin{pmatrix} 16 \\ 8 \end{pmatrix}$

   $= 8 \times \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

   Since $\mathbf{x} + 3\mathbf{y}$ is a multiple of $\mathbf{z}$, they are parallel

   b -8

12 a 343.4°   b 5.1